LABORATORY 10: Stress Analysis

I. Stress Field Ellipsoid

A) Any state of stress can be fully represented by the *stress ellipsoid*. The stress ellipsoid is a triaxial ellipsoid that is defined by three axes of different length:

1. σ_1: maximum *normal stress* axis.
2. σ_2: intermediate normal stress axis.
3. σ_3: minimum normal stress axis.

For conditions of *lithostatic stress*, the above stress axes are equal in magnitude and therefore no plane that passes through a body subjected to a lithostatic stress will have a shear stress.

B) Note that all three axes are mutually perpendicular. Also there are unique directions of normal stress for σ_1 and σ_3 (maximum and minimum normal stress) but there are an infinite number of normal stress directions equal to σ_2 that are arrayed in the two *circular sections* of the stress ellipsoid. The total condition of stress affecting a rock mass can be described as the *stress tensor*, which is a second-order tensor. An single instance of a force acting on an area can be described as a stress traction. Both the stress field tensor and stress tractions are second-order tensors, which means that they cannot be resolved as simple vector quantities.

C) A plane that passes through a body under a stress field that is also perpendicular to one of the principal normal stress directions will have no *shear stress (τ)* on it. Any other plane will have some value of shear stress upon it.

D) The stress ellipsoid is extremely useful in predicting fault and joint formation in real rocks under a stress field. A graph termed the *Mohr Circle Diagram* has been developed to determine the magnitude of normal stress and *shear stress* on any plane that passes through the rock mass subjected to a *stress tensor*. The orientation of the principal axes of the stress ellipsoid and the fracture planes can be tracked on the stereonet.

E) The development of a fracture plane to form a fault or joint will occur in a rock mass when the ratio of τ/σ reaches a critical value. The locus of these critical values is termed the *Mohr fracture* envelope that can be empirically determined for any rock by experimental means.

II. Mohr Circle Diagram

A) The Mohr circle is a circle plotted on a (X, Y) graph defined by normal stress values (σ) along the x axis and shear stress (τ) along the y axis. The diameter of the Mohr circle is defined by the position of σ_1 and σ_3 on the x axis. Note that σ_2 is essentially ignored on this diagram since fractures are parallel to the intermediate axis.

B) The below diagram is an example of a Mohr circle graph with the fracture envelope plotted. The two fracture planes labeled A and B are termed the *conjugate fractures* because, if the rock is mechanically homogenous, the two fractures will form simultaneously and symmetrically about the maximum normal stress direction.

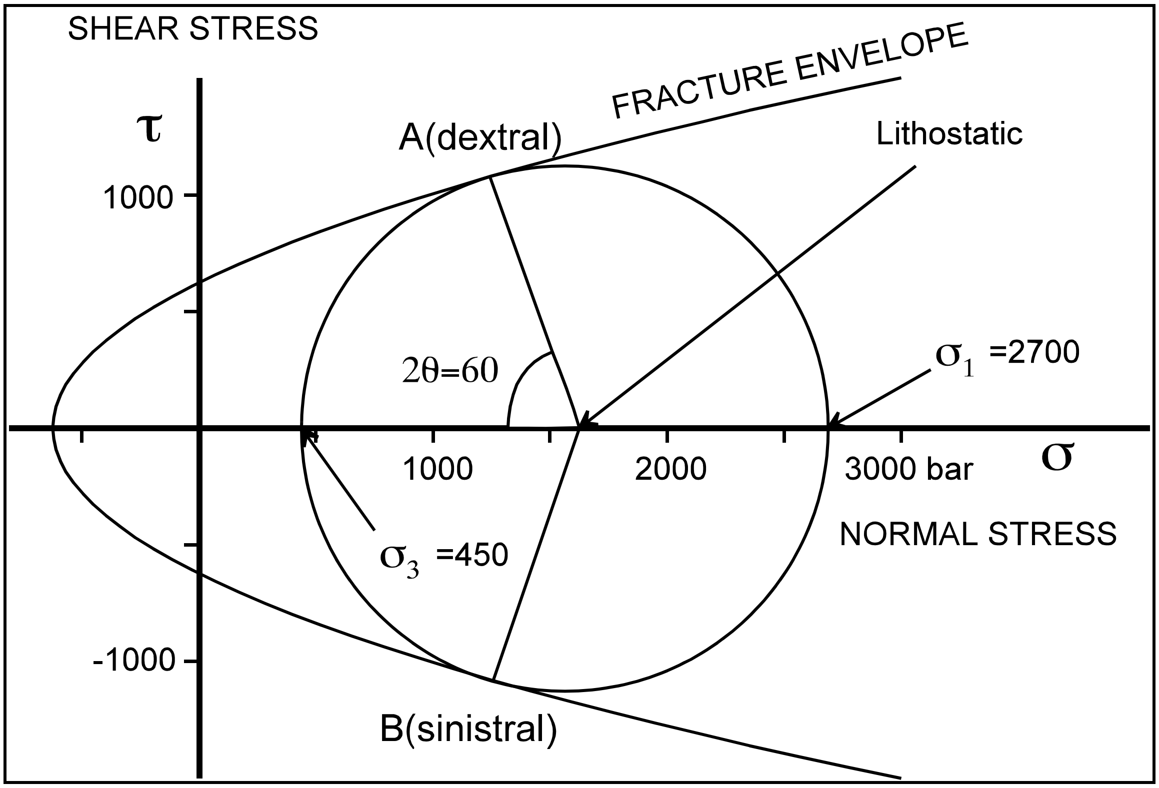

Figure 10-1: Example of the Mohr stress circle with fracture envelop.

C) The perimeter of the circle represents all of the possible stress states on planes passing through the center of the stress field. The center of the circle is fixed by the *lithostatic stress* value which is dependent on the burial depth. The angle along the perimeter of the circle from σ_3 to the point of interest is termed the 2θ angle. For fracture (A), 2θ is +60°, whereas for fracture plane (B), 2θ is -60°.

D) The conjugate fracture planes (A) and (B) have equivalent σ and τ magnitudes, however, (A) has dextral (+) sense of shear, while (B) has sinistral (-) sense of shear. Also note that on the Mohr Circle diagram in Figure 10-1 that the 2 theta (2θ) angle is measured clockwise from σ_3 for positive values of τ.

E) The below Figure 10-1 diagram displays the relationship of the principle stresses and shear planes with respect to an actual physical specimen that is deformed under laboratory conditions. Note that the shear plane has a dextral sense of shear and therefore a positive value of τ. This means that if the angle measured from the shear plane to σ_1 is clockwise, then τ is positive and the sense of shear must be dextral. Likewise, an anticlockwise angle measured from the fracture plane to σ_1 defines a negative τ shear plane. Remember that this rule is used when viewing the actual physical sample. Also noteworthy is the fact that within in the physical specimen if two conjugate shear planes from, that σ_1 will bisect the acute angle and σ_3 the obtuse angle.

III. Constructing the Mohr Circle Graph

A) The construction of the Mohr Circle graph assumes that σ is plotted on the X axis, and τ on the Y axis. This is normally done on standard 10sqi graph paper, or alternatively with the charting capabilities of a spreadsheet application. If the graph is plotted manually, care should be taken selecting the range of the X and Y axes to ensure that the Mohr Circle will fit on the sheet of paper. It is usually necessary to place the Y axis at sigma values greater than 0 because sigma may be large compared to tau. The scale of both axes should be equivalent (i.e. one inch = 200 bar).

B) If the maximum and minimum values of σ are known, the Mohr circle can be plotted by constructing a circle that passes through those points on the x axis, and which has a center halfway between the two points. Mathematically the center of the circle is $(\sigma_1 + \sigma_3)/2$.

C) If the stress state of two planes that pass through the stress field are known (σ and τ), these two points must fall on the perimeter of the Mohr circle. The perpendicular bisector of the chord between these two points will cross the x axis at the center of the circle.

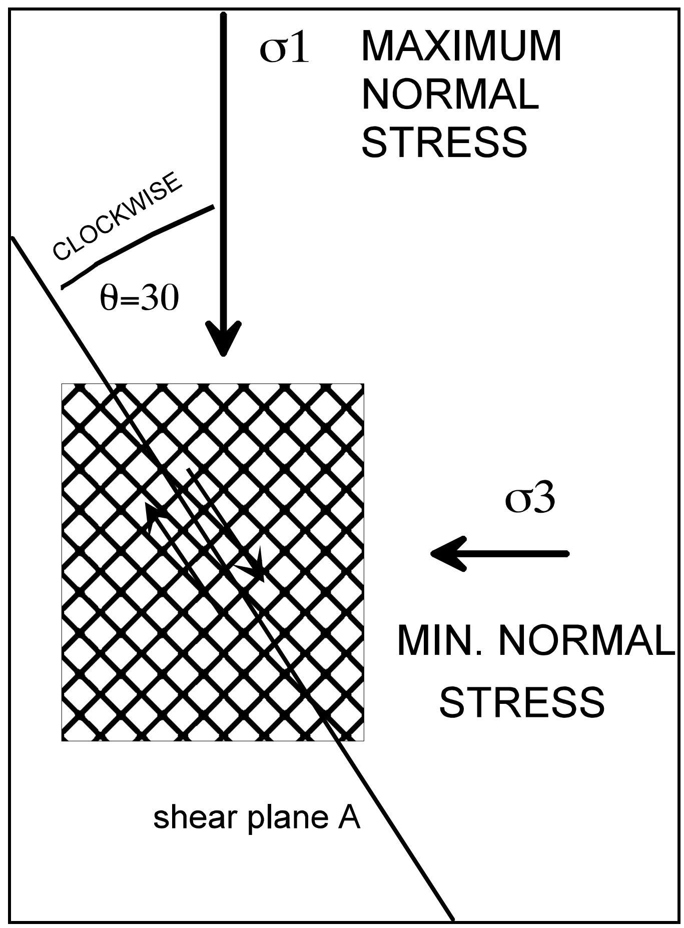

Figure 10-2: Actual physical test specimen for Mohr circle example.

IV. Determining the Attitude of Stress Axes and Fracture Planes

A) In addition to the magnitude of the principal stress axes, the geologist must also relate the orientation of the stress ellipsoid to a "real-world" coordinate system. The most convenient device for accomplishing this is the stereonet with geographic north indexed in the standard way.

B) The following rules should serve as guidelines for plotting stress filed elements:

1. Remember that the three principal stress ellipsoid axes are mutually perpendicular, therefore, if the attitude of any two of the axes are determined the other third must be located 90° to the plane that contains the other two.

2. Conjugate shear planes always intersect at the intermediate normal stress axis σ_2 axis.

3. The maximum principle stress, σ_1, bisects the acute angle formed by the conjugate shear planes (see Figure 10-2). Likewise, the minimum principle stress σ_3 must bisect the obtuse angle formed by the conjugate shear planes.

4.If the 2θ is measured from the Mohr circle plot, remember that in reality, θ is the angle that the fracture plane makes with σ_1. A common mistake is to use θ as if it where the angle that the fracture plane makes with the minimum normal stress direction (σ_3) because that is the reference from which 2θ is measured on the Mohr stress circle.

5. To determine the sense of shear on a fracture plane plotted on the stereonet it may be necessary to rotate the entire diagram until the σ_1-σ_3 is horizontal.

C) Remember that the Mohr circle graph can determine magnitudes of stress for any plane passing through the stress field, and it can determine a 2θ value. It cannot, however, yield any information about the orientation of the stress ellipsoid axes or the fracture planes.

D) The stereonet can solve for the attitude of the stress ellipsoid axes and shear planes, however, it cannot yield any information on the magnitude of the stresses.

V. Mathematical Basis for Mohr Circle

A) The Mohr circle can easily be expressed as a function of the maximum and minimum normal stress values. With the below equations the stress state of any plane passing through the stress field can be calculated:

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2}\cos(2\theta)$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2}\sin(2\theta)$$

where σ and τ represent the normal and shear stress respectively acting on the plane of interest subject to a stress tensor of magnitudes σ_1 and σ_3. The angle θ is the angle that the plane of interest makes with σ_1 measured clockwise from the plane to σ_1. Furthermore, θ must be less than 90° in absolute magnitude, therefore, if the angle less than 90° is measured counterclockwise from the plane to the σ_1 direction, then θ is negative.

EXERCISE 10: Mohr Circle and Stress Calculations

Problem 1. Given the following orientations for the principal stress axes:

σ_1 : 0°, N90°E
σ_3 : 90°, N0°E

find the orientation of σ_2. If the θ angle is 20° for conjugate fractures, plot both shear planes on the stereonet along with the principal normal stress directions (σ_1, σ_2, and σ_3) labeled. As observed from the south looking to the north, label the conjugate shear planes in terms of the sense of shear movement- sinistral or dextral. If the conjugate shears are considered to be faults, how would you classify each fault? Determine and report the attitude of each shear plane.

Problem 2. Assume that:

σ_1 = 2050 bars 50°, N40°E
σ_3 = 1600 bars 24°, S18°E

Scale: 1"=200 bars

What is the maximum value of τ (shear stress) along any plane that passes through a body under the above stress state (determine graphically)? What is the orientation of σ_2 (determine on the stereonet)? What are the orientations of the body planes that have maximum $\pm\ \tau$ values (determine on the stereonet)? Report the sense of shear for each plane with your answer.

Problem 3. Given the following values of σ and τ stress for two planes passing through a body under a stress field:

	σ	τ	Orientation
1.	1800	100	N0°E, 50°W
2.	2100	200	N64°E, 30°NW

Find the values of σ_1 and σ_3 (graphically). On a stereonet plot the orientation of the two planes, along with the orientations of σ_1, σ_2, and σ_3. Report the values of 2θ for each shear plane
Scale: 1"= 200 bars.