LABORATORY 11: Strain Analysis

I. Strain Analysis

A) *Strain marker*s must be present in rock before strain can be analyzed.

B) There must be little or no mechanical difference between strain marker and matrix, otherwise the strain analysis may be invalid.

Examples of low mechanical contrast
1. Oolites in limestone
2. Quartz pebbles in a quartzite (Metaconglomerate)

C) In practice most strain markers are not perfectly spherical in the undeformed state- if they were we could directly measure the dimensions of any individual ellipsoid to obtain the *finite strain ellipse* dimensions.

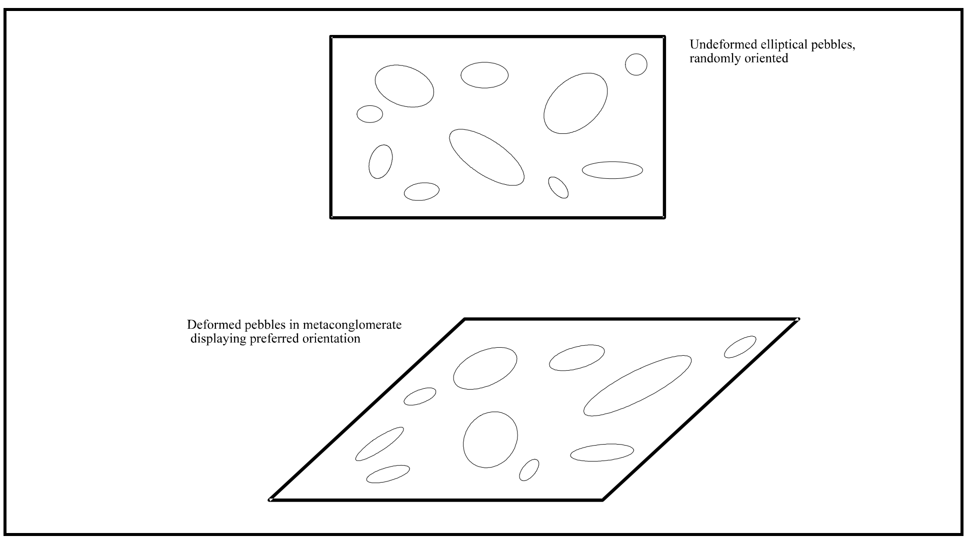

Figure 11-1: Simple shear of initially random ellipsoidal pebbles to form a preferred orientation of strain ellipsoids.

D) Field analysis of deformed rocks generally assumes *homogenous plane strain* (constant volume conditions) because it greatly simplifies calculations and is valid if the *dilation* component of strain is not significant.

E) We assume that strain markers began as randomly oriented ellipsoidal objects before deformation, such as pebbles in a stream bed. If strain is truly homogenous the original *ellipsoid*s will still be ellipsoidal after deformation. The ellipsoids will display a dimensional preferred orientation after deformation. Note that non-random original orientations will still produce a preferred attitude after deformation, however, it may invalidate analysis of strain using the hyperbolic net, and the R_F/Φ method described below.

F) *Φ* is the measured angle that the X axis of an elliptical strain marker makes with some reference direction. Usually this direction is chosen to be a significant tectonite direction, such as the trace of S_1 at the exposure. In addition to Φ, the axial ratio (X/Z) is measured as the R_F value for each ellipse.

G) The *finite strain ellipse* should be imagined to be the ellipsoid body that result from the deformation of an original sphere with diameter equal to 1.0 length. This body is also imagined to have suffered all deformation affecting the rocks under consideration. Usually one of the goals of kinematic analysis is to find the dimensions and attitudes of the axes of the finite strain ellipse. The ratio of the X and Z axes of the finite strain ellipse (X/Z) is referred to as R_S. Likewise, the angle that the finite strain ellipse makes with the chosen reference direction is termed Φ_S.

II. Use of the Hyperbolic Net (De Paor's Method)

A) After R_F and Φ values are tabulated for all strain marker ellipses, they are plotted on a special projection termed the *hyperbolic net*. The goal of this procedure is to determine the R_S and Φ_S values for the total finite strain ellipse. The hyperbolic net uses the relationships demonstrated in Figure 11-1 above to define the "best-fit" hyperbolic curve relative to the data.

B) Data is plotted on the hyperbolic net after the Φ and R_F values are tabulated:

1. Rotate the overlay so that the Φ angle position is at the due north point of the primitive. Positive angles are to the clockwise of the "R" reference tic, negative angles are counterclockwise.

2. From the center of the net move up along the vertical line until the hyperbolic curve that corresponds to the value of R_F is found. Mark a dot at this point. Note that the center of the net begins at a value of 1.

3. Continue with steps (1) and (2) above until all data is plotted.

C) After plotting the data on the net, follow these steps:

1. Sketch a smooth line around all of the data points. The polygon thus formed should be as “simple” as possible (i.e. lowest number of sides).

2. Find the north-south line on the overlay that divides the data distribution into equal area halves. This line defines the Φ_S direction angle.

2. Find the “best-fit” hyperbolic curve that splits the data into 25% area quarters. This hyperbolic curve defines the value of the axial ratio (X/Z) for the finite strain

ellipse. This value is R_S. After drawing this curve on the overlay, the four quadrants defined by the Φ_S and R_S curves should divide the data into roughly 25% proportions.

D) Note that if the data deviate significantly from the 25% per quadrant rule the strain markers probably had a preferred attitude before deformations. This may invalidate the R_F/Φ method.

E) If we assume plane strain, and, therefore, constant volume throughout deformation, we can calculate the actual dimensions of the finite strain ellipsoid assuming a convenient pre-deformation diameter, such as 1 unit length. This in turn allows the calculation of stretch (S) values in the principle directions. Once these values are determined, we can calculate, using the general Mohr circle strain equations, the values of λ, γ, Ψ, and α for any direction defined as θ_d.

III. Plotting the Attitude of the Finite Strain Ellipse

A) The X, Y, and Z axes of the finite strain ellipsoid are mutually perpendicular, therefore, any one of these axes will be the pole to the plane defined by the other two axes. Since geologists usually purposely find the X-Z plane and measure the attitude of this plane at the exposure, usually the Y axis is assumed to be the pole to the XZ plane.

B) Oriented sample must be taken and labeled in the field if one is to calculate the attitude of the finite strain axes. In the field this is done by physically drawing the strike and dip lines on the X-Z surface of the sample before it is disturbed. In this way the sample may be re-oriented in the laboratory.

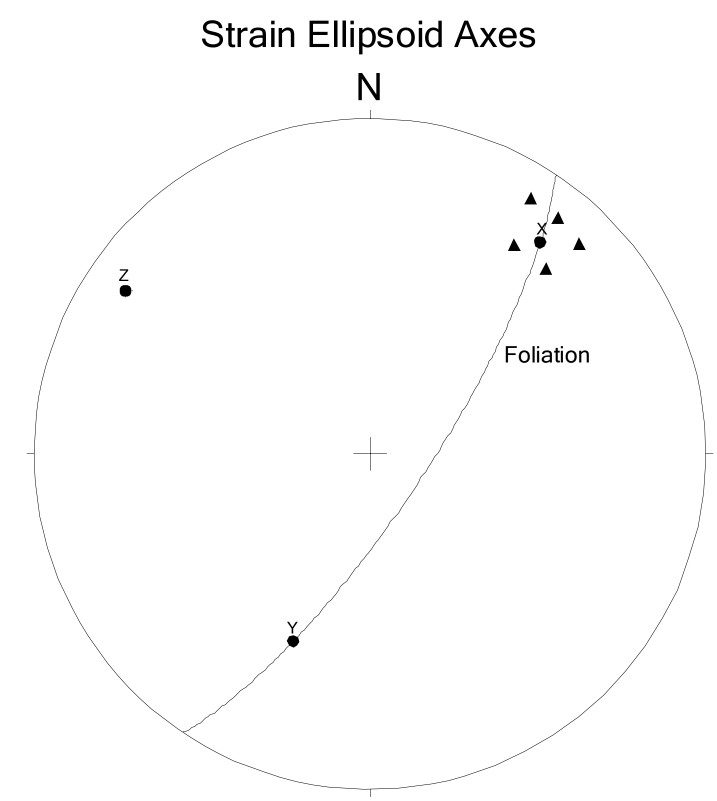

Figure 11-2: Plot of strain axes and foliation.

C) It is often possible to relate the finite strain ellipse to tectonite fabric elements such as S_1 foliation. Stretching lineations tend to develop parallel or sub-parallel to the X axis, and are oriented in the X-Y plane. The S1 foliation plane is often equivalent to the X-Y plane, therefore, plots of lineation data may fall systematically along the S_1 great circle. Alternatively, plots of poles to S_1 will plot in the vicinity of the minimum elongation (X) of the finite strain ellipse.

IV. Solving for the Dimensions of the Finite Strain Ellipse

A) If plane strain is assumed, we know that the starting reference sphere and the resulting finite ellipsoid will have the same volume.

B) Mathematical proof:

Given:

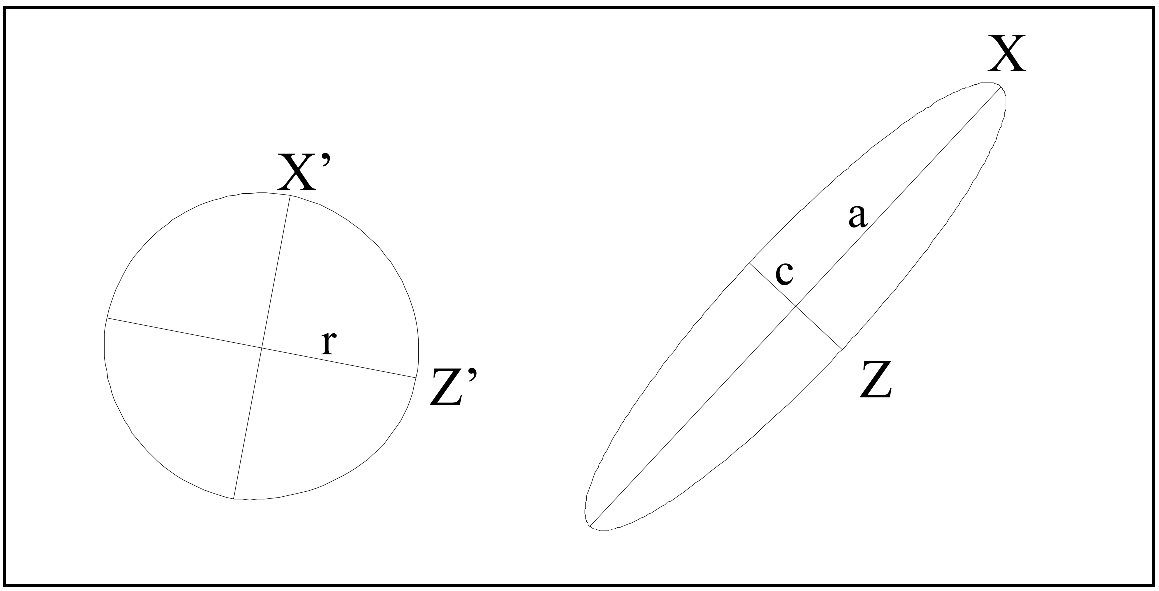

Figure 11-3: Undeformed and deformed strain marker reference used for derivation of formulae.

V_i = volume of undeformed sphere with radius r.
r = 1.0
V_f = volume of final ellipsoid that is the product of the homogenous deformation of the initial sphere of volume V_i. The (X, Y, Z) directions of the ellipsoid are parallel to the (a, b, c) directions in the below figure:

R_S = ratio of X to Z for the finite strain ellipse.

Find: Dimensions of the total finite strain ellipsoid assuming that the original sphere is deformed by plane strain. Under these conditions the initial sphere and final ellipsoid should have equal volume:

$V_i = V_f$
$V_i = (4/3)\pi r^3$
$V_F = (4/3)\pi abc$

Setting both volume equations equal to one another we can simplify to:

$r^3 = abc$

Because the deformation is via plane strain, the length of b should equal the original length r:

$r^2 = ac$

In the original definition of the proof r = 1, therefore:

$1 = ac$

Now we can use the definition of R_S to solve simultaneous equations:

$R_S = X/Z = 2a/2c = a/c$

Substituting into the above equation:

$a = R_S\ c$
$1.0 = (R_S)(c)(c)$
$1.0 = R_S\ c^2$
$c = \mathrm{Sqrt}(1/R_S)$

Therefore:

$a = 1/c = \mathrm{Sqrt}(R_S)$

With these equations you can convert the R_S value measured from the hyperbolic net directly into the dimensions of the finite strain ellipse. From the dimensions the axial stretch values can be calculated:

$S_X = a/r = a = \mathrm{Sqrt}(R_S)$
$S_Z = c/r = c = \mathrm{Sqrt}(1/R_S)$

EXERCISE 11: Strain Analysis

Problem 1

In Figure 11-4 is a photograph of deformed ooids in limestone. Assuming that the ooids have been affected by homogenous plane strain, conduct a strain analysis using the hyperbolic net (De Paor's method). Use the traced outlines of the ellipsoids in Figure 11-5 to measure R_F and Φ. Measure Φ relative to the N32W reference line in Figure 11-5. Report your measurements with a table organized as follows:

No.	X(inches)	Z(inches)	R_F	Φ
1	0.822	0.590	1.393	-4
2	0.665	0.481	1.383	6
.	.	.	.	.
.	.	.	.	.

After finding R_S and Φ_S, your goal will be to find the dimensions of the strain ellipsoid (i.e. the total finite strain ellipsoid) that before deformation is assumed to have been perfect sphere with a diameter of 1.0. When the dimensions are known, calculate the S (stretch value) for the X and Z directions of the total finite strain ellipse. Remember that if you can assume homogenous plane strain, then you can also assume that each ooid maintains constant volume in three dimensions and area in two dimensions before and after deformation. Assuming that the surface in Figure 11-5 approximates the X-Z plane of the strain ellipsoid, and that foliation is perpendicular to this surface, plot the (X, Y, Z) directions of the total finite strain ellipse on the stereonet. Assume that the reference line in Figure 11-5 is the strike of the foliation , and that the dip of the foliation is 25SW. Plot and label the foliation plane and the X-Z plane on the stereonet as great circles

Problem 2

In Figure 11-6 are the traced and numbered outlines of deformed pebbles in a sample (CA-23) of the Cheaha Quartzite. At the outcrop where this sample was collected the foliation attitude was N10E, 35SE. In Figure 11-6, two parallel sides of the slabbed sample are traced so that you can measure the X/Z ratios on both sides. Note that the trace line of the S_1 foliation is to be used as the Φ reference line on both faces of the sample. The sides of the slabbed sample are to be assumed to be cut perpendicular to S_1 foliation. In addition, you are to assume that the trace of S_1 on both sides of the slab in Figure 11-6 are parallel to the strike line (N10E) of foliation. Use the hyperbolic net to calculate R_S and Φ_S for the deformed pebbles. Use a table organized as in Problem 1 to report your measurements. From those measured values, calculate the dimensions of the total finite strain ellipse assuming plane strain and a starting reference sphere with diameter equal to 1.0. Calculate the stretch values parallel to the X and Z directions, and plot the (X, Y, Z) axis attitudes of the finite strain ellipse on the stereonet. Also plot the “slab plane” and the foliation lane as great circles, and label them on the stereonet. Calculate the values of λ, S, γ, Ψ, and α for a line in Figure 11-6 parallel to the long axis of pebble #25.

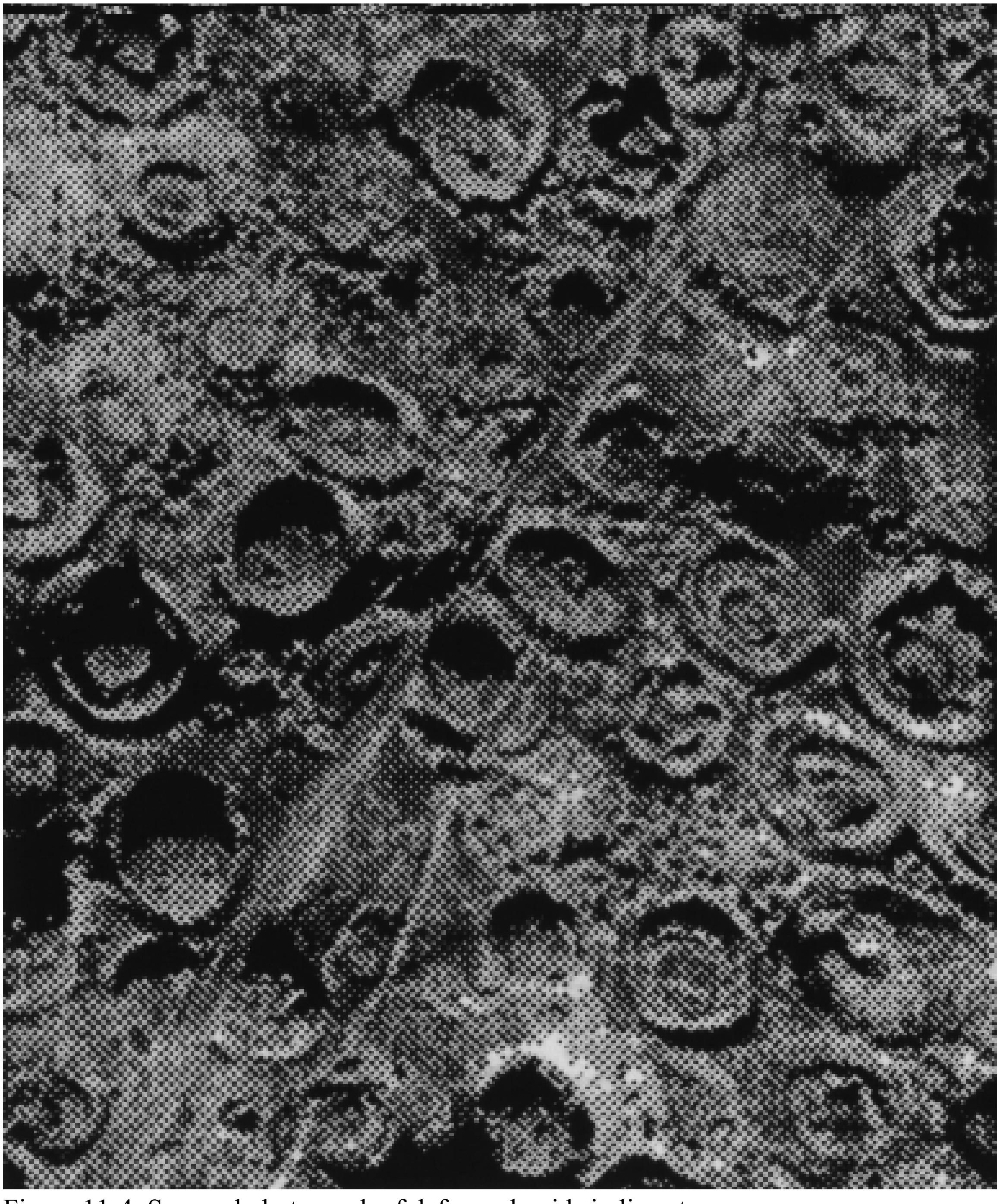

Figure 11-4: Scanned photograph of deformed ooids in limestone.

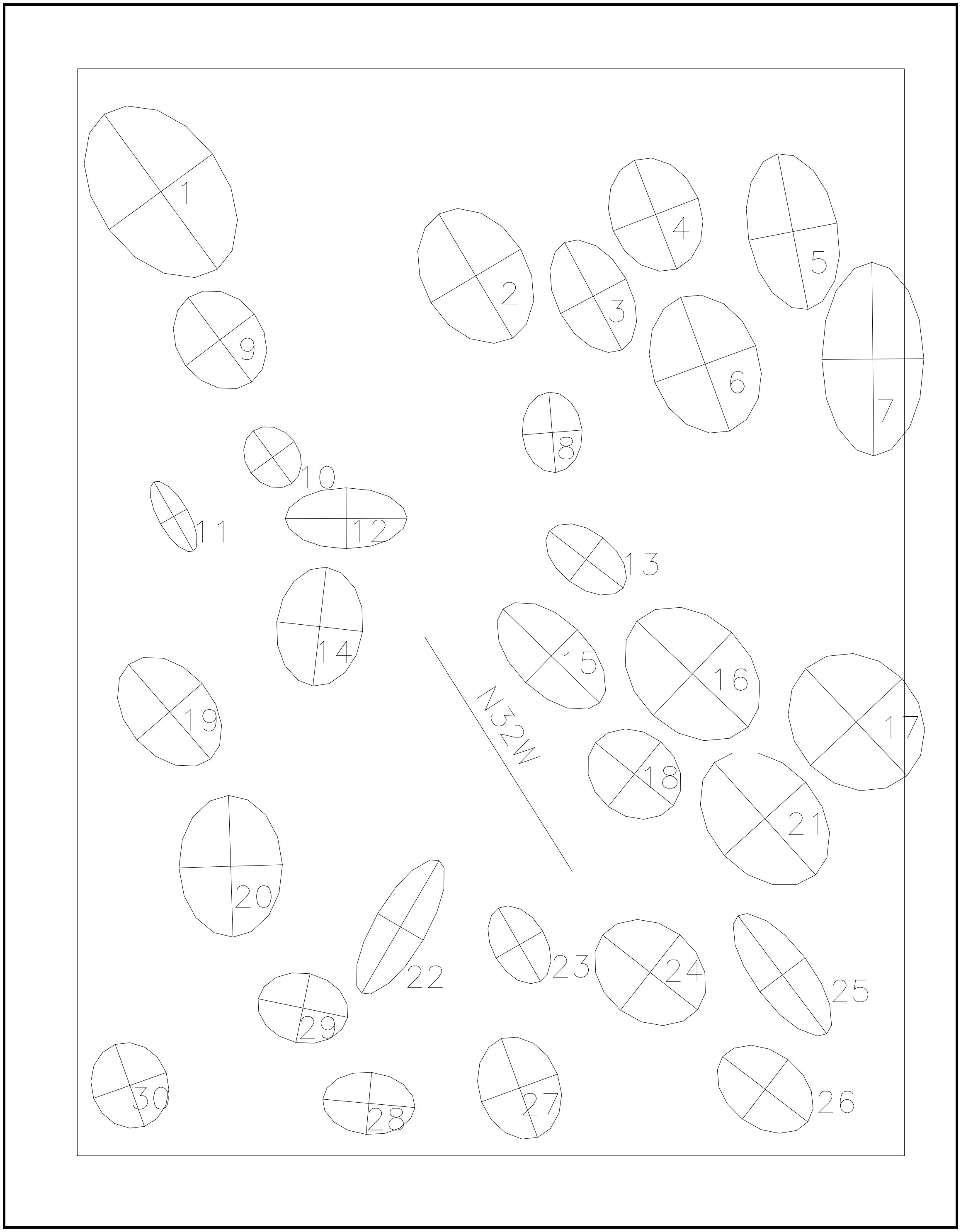

Figure 11-5: Tracing of the deformed ooids in Figure 11-4. Use this to calculate R_F and Φ.

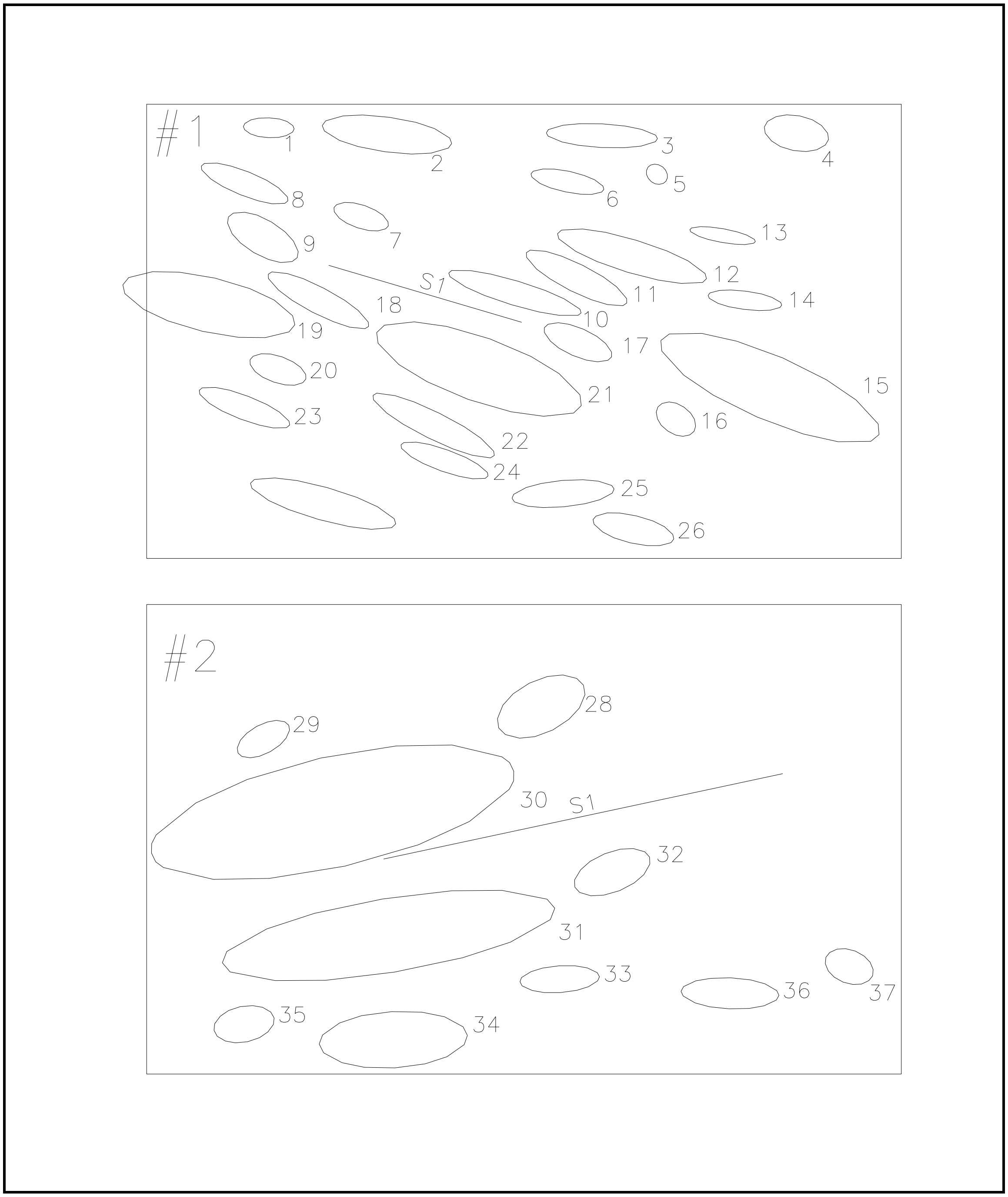

Figure 11-6: Tracing of deformed pebbles in Cheaha Quartzite. Two parallel faces of the same sample (CA-23) are displayed.

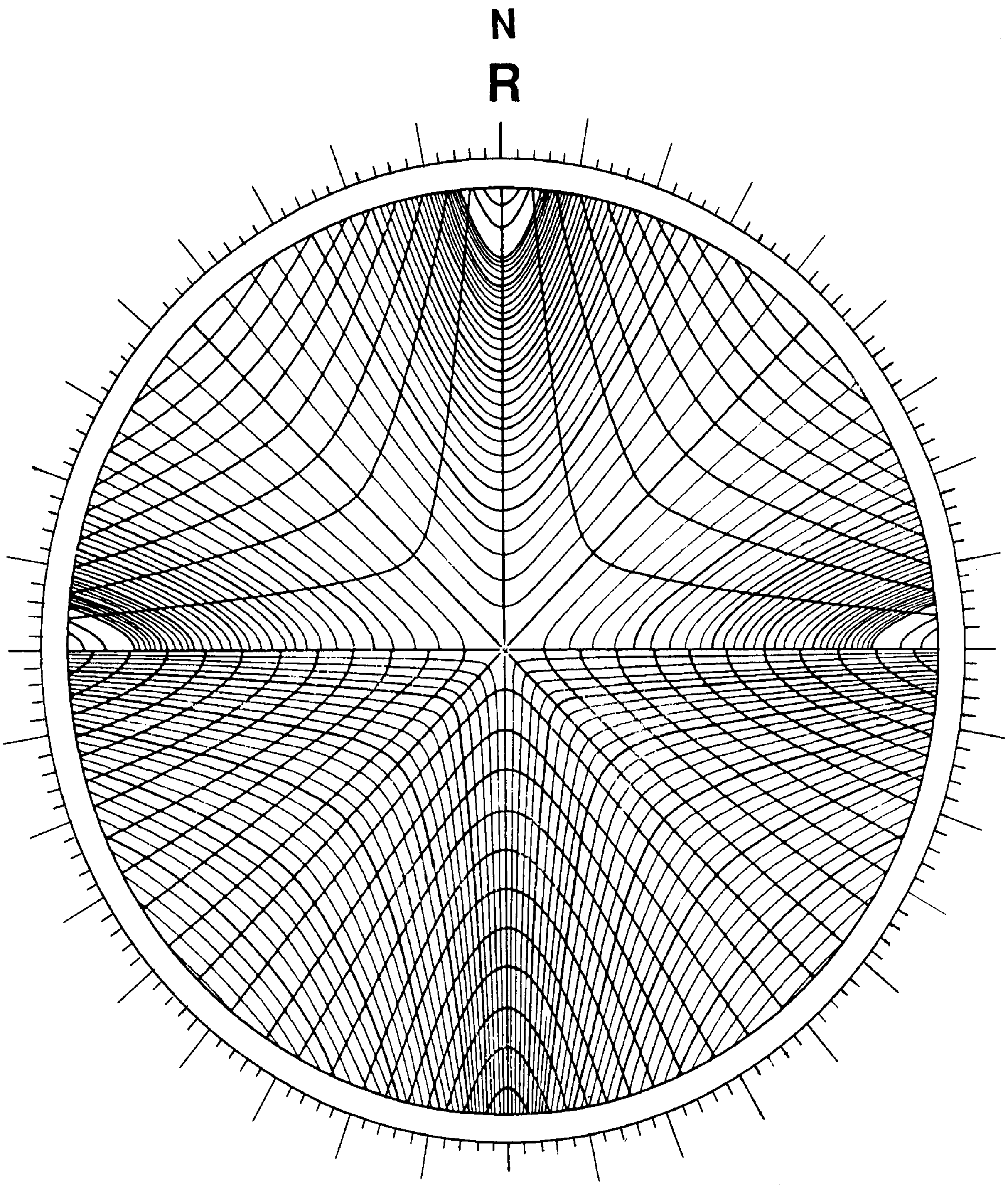

Figure 11-7: Hyperbolic stereonet.