

LABORATORY 9: Stereographic Statistical Techniques

I. Introduction

a) Statistics can be used with the stereonet as a predictive tool. Usually the structural geologist is most interested in patterns on the stereonet that indicate a type of structure, such as a fold. In addition statistics can define a *confidence region* on the stereonet that contains a probability value that predicts how likely future measurements would fall inside the region.

b) There are several types of geometric patterns that are indicative of structures:

1. *Point cluster*: a cluster of points indicate that most of the data have approximately the same attitude. This is true of linear data or poles to planes. Solving for the *least-squares* or *best-fit vector* to the data set gives the "center of gravity" of the data (i.e. *mean vector*)

2. *Cylindrical girdle*: points that are aligned along a great circle represent vectors that are contained within a common plane in three dimensions. Since poles to a folded surface, such as bedding, have this property this type of distribution is termed cylindrical because the folded surface is approximated by a section of a cylinder. The pole to the girdle plane is the hinge attitude of the fold. The girdle is also the great circle arc along which the *interlimb angle* can be measured since it represents the plane perpendicular to the hinge. A statistical least-squares solution will yield the attitude of a geometric plane that minimizes the deviations of the data from the girdle great circle.

3. *Conical distribution*: a conical distribution is representative of vectors which fall along a small circle. These distributions can be produced by several different mechanisms. Some folds are not cylindrical but are instead inherently conical in shape. Conical folds will "die out" along the axial trace. Originally cylindrical folds may become conical after being re-folded by later deformation. Lineations that exist in a rock mass that is later affected by cylindrical *parallel folding* will be deformed into a small circle distribution if, as is likely, their original attitude was not perpendicular to the later fold axis. Solving for the least-squares conical surface for data that has been affected by this type of deformation yields a conical axis and a *K angle*. The K angle is also termed the $\frac{1}{2}$ *apical angle*. This is the angular arc from the cone axis to the least-squares conical surface.

c) The mathematical equations that calculate statistical parameters must use data in the form of *directional cosines*. The attitude of geometrical least-squares elements are also solved for in the form of directional cosines. Note that any directional cosine can be checked for validity by the following relationship:

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1.0$$

where α , β , and γ are the directional angles of the vector.

II. Least-squares Vector of Ramsay (1968)

a) Equations for the least-squares vector solve for the directional cosines of the vector:

$$F = \sqrt{\sum_{i=1}^n [(\cos \alpha_i)^2 + (\cos \beta_i)^2 + (\cos \gamma_i)^2]}$$

where α_i , β_i , and γ_i are the directional angles for the data set with summation from $i = 1$ to the “ n th” data element.

$$\cos(\alpha_V) = \frac{\sum_{i=1}^n \cos(\alpha_i)}{F} \quad \cos(\beta_V) = \frac{\sum_{i=1}^n \cos(\beta_i)}{F} \quad \cos(\gamma_V) = \frac{\sum_{i=1}^n \cos(\gamma_i)}{F}$$

where $[\alpha_V, \beta_V, \gamma_V]$ represent the attitude of the least-squares vector.

III. Least-squares Cylindrical Plane of Ramsay (1968)

a) Equations for the least-squares cylindrical plane are necessarily more complex than those for the vector, therefore, to simplify the below equations let:

$$\begin{aligned} l &= \cos(\alpha_i) \\ m &= \cos(\beta_i) \\ n &= \cos(\gamma_i) \end{aligned}$$

therefore, whenever $[l, m, n]$ are present in the below formulae they actually represent $[\cos(\alpha), \cos(\beta), \cos(\gamma)]$ for the “ i th” data vector element respectively:

$$\text{let } E = \Sigma(l^2)\Sigma(m^2) - \Sigma((lm)^2)$$

$$A = \frac{\sum (lm)\sum (mn) - \sum (nl)\sum (m^2)}{E}$$

$$B = \frac{\sum (lm)\sum (nl) - \sum (mn)\sum (l^2)}{E}$$

$$C = \frac{A}{\sqrt{1 + A^2 + B^2}}$$

$$\cos(\alpha_p) = \frac{A}{\sqrt{1 + A^2 + B^2}}$$

$$\cos(\beta_p) = \frac{B}{\sqrt{1 + A^2 + B^2}}$$

$$\cos(\gamma_p) = \frac{1}{\sqrt{1 + A^2 + B^2}}$$

where $[\alpha_p, \beta_p, \gamma_p]$ represent the directional angles of the hinge of the cylindrical fold, which is also the pole to the least-squares plane. All of the summation symbols above are for $i = 1$ to n data. This is also true for all following summation symbols.

IV. Least-squares Conical Surface of Ramsay (1968)

a) Solving for the least-squares conical surface requires the extraction of determinants from the below matrices. The notation of (l, m, n) is equivalent to that used in Ramsay's method for a cylindrical fit. "N" is equivalent to the number of data observations:

$$D = \begin{matrix} & \Sigma(l^2) & \Sigma(lm) & \Sigma(l) \\ D = & \Sigma(lm) & \Sigma(m^2) & \Sigma(m) \\ & \Sigma(l) & \Sigma(m) & N \end{matrix}$$

$$D_A = \begin{matrix} & -\Sigma(ln) & \Sigma(lm) & \Sigma(l) \\ D_A = & -\Sigma(mn) & \Sigma(m^2) & \Sigma(m) \\ & -\Sigma(n) & \Sigma(m) & N \end{matrix}$$

$$D_B = \begin{matrix} & \Sigma(l^2) & -\Sigma(ln) & \Sigma(l) \\ D_B = & \Sigma(lm) & -\Sigma(mn) & \Sigma(m) \\ & \Sigma(l) & -\Sigma(n) & N \end{matrix}$$

$$D_C = \begin{matrix} & \Sigma(l^2) & \Sigma(lm) & -\Sigma(ln) \\ D_C = & \Sigma(lm) & \Sigma(m^2) & -\Sigma(mn) \\ & \Sigma(l) & \Sigma(m) & -\Sigma(n) \end{matrix}$$

NOTE: most current computer spreadsheet programs have a function that extracts the determinant from a square matrix of values. For example, Quattro for Windows has a @MDET(range) function where "range" would represent the cell range of the diagonal cells of the matrix.

$$D = \Sigma(l^2)\Sigma(m^2)(N) + \Sigma(lm)\Sigma(m)\Sigma(l) + \Sigma(l)\Sigma(lm)\Sigma(m) \\ - \Sigma(l)\Sigma(m^2)\Sigma(l) - \Sigma(lm)\Sigma(lm)(N) - \Sigma(l^2)\Sigma(m)\Sigma(m)$$

$$D_A = -\Sigma(ln)\Sigma(m^2)(N) + \Sigma(lm)\Sigma(m)(-\Sigma(n)) + \Sigma(l)(-\Sigma(mn))\Sigma(m) \\ - \Sigma(l)\Sigma(m^2)(-\Sigma(n)) - \Sigma(lm)(-\Sigma(mn))(N) - (-\Sigma(ln))\Sigma(m)\Sigma(m)$$

$$D_B = \Sigma(l^2)(-\Sigma(mn))(N) + (-\Sigma(ln))\Sigma(m)\Sigma(l) + \Sigma(l)\Sigma(lm)(-\Sigma(n)) \\ - \Sigma(l)(-\Sigma(mn))\Sigma(l) - (-\Sigma(ln))\Sigma(lm)(N) - \Sigma(l^2)\Sigma(m)(-\Sigma(n))$$

$$D_C = \Sigma(l^2)\Sigma(m^2)(-\Sigma(n)) + \Sigma(lm)(-\Sigma(mn))\Sigma(l) + (-\Sigma(ln))\Sigma(lm)\Sigma(m) \\ - (-\Sigma(ln))\Sigma(m^2)\Sigma(l) - \Sigma(lm)\Sigma(lm)(-\Sigma(n)) - \Sigma(l^2)(-\Sigma(mn))\Sigma(m)$$

where (l, m, n) have the same symbolic meaning as in the previous discussion. (N) refers to the number of data. From the determinants the following coefficients may be calculated:

$$A = \frac{D_A}{D} \quad B = \frac{D_B}{D} \quad C = \frac{D_C}{D}$$

and from these coefficients the directional cosines are calculated:

$$\text{Cos}(\alpha_C) = \frac{A}{\sqrt{1 + A^2 + B^2}}$$

$$\text{Cos}(\beta_C) = \frac{B}{\sqrt{1 + A^2 + B^2}}$$

$$\text{Cos}(\gamma_C) = \frac{1}{\sqrt{1 + A^2 + B^2}}$$

$$\text{Cos}(K_C) = \frac{-C}{\sqrt{1 + A^2 + B^2}}$$

where $[\alpha_C, \beta_C, \gamma_C]$ represent the directional angles of the conical axis, and K_C is the $\frac{1}{2}$ apical angle of the least-squares conical surface.

V. Eigen Vectors

a) Eigenvectors are mathematically calculated using matrix algebra in a way that is

different than the Ramsay (1968) procedures described before. The eigenvectors are mutually perpendicular in three dimensions, and are related to the mean attitude of the structure data set.

B) The eigenvectors are also the three axes of an ellipsoid. This ellipsoid should be imagined as the best-fit surface to the data set if each data is a vector of unit length (either linear structure elements or poles to planes), and the surface best-fits the end points of the vector. The midpoint of each unit vector would be at the center of the

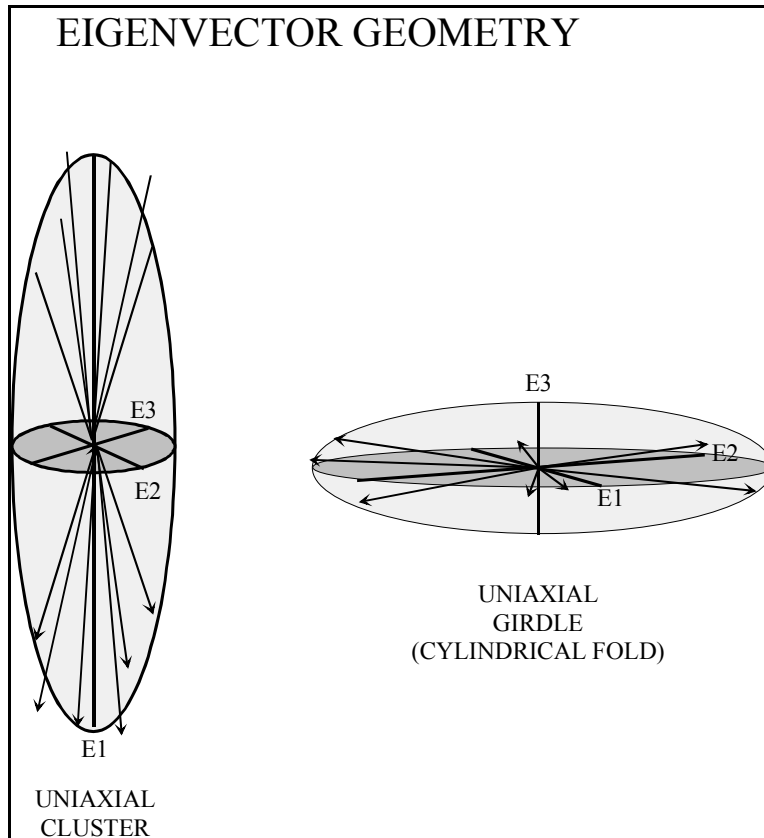


Figure 9-1: Examples of eigenvector axial lengths.

stereographic projection sphere. In this regard, the eigenvector method makes no distinction between the upper and lower hemisphere as does the Ramsay method. Thus, the three eigenvectors are the three axes of the best-fit ellipsoid. Vector clusters produce a prolate ellipsoid, while cylindrical fold distributions yield oblate ellipsoids.

C) The Ramsay method suffers from the “double-plunge” effect because of the lower-hemisphere projection. For example, if an equal number of low-plunge lineations are doubly plunging north and south, the calculated average vector using the Ramsay method yields a vertical mean vector! The eigenvector method instead yields the correct horizontal north-south oriented vector.

D) To calculate the eigenvectors, the structure data must be converted into directional cosines, with which the below summations can be made:

$$\begin{aligned}\cos(\alpha) &= l \\ \cos(\beta) &= m \\ \cos(\gamma) &= n\end{aligned}$$

	l	m	n	“check”	lm	ln	mn	l ²	m ²	n ²
Data #1										
Data #2										
.										
.										
.										
Data #N										

Summations:

When the summations are accumulated, the values can be processed by an eigenvector procedure to derive the magnitude and attitude of the three eigenvectors. Note that while the eigenvector 3x3 matrix contains 9 summation elements, some are equivalent, therefore, only 6 summations are actually required:

$$A = \begin{matrix} \sum l^2 & \sum lm & \sum ln \\ \sum ml & \sum m^2 & \sum mn \\ \sum nl & \sum nm & \sum n^2 \end{matrix}$$

Where “A” is the eigenvector solution. The results are best categorized graphically by Figure 9-1 (Woodcock, 1977). The results from this graph fall in to one of three categories:

1. **Uniaxial Cluster:** this distribution indicates that data varies only slightly in attitude. A typical example would be mineral lineations in a metamorphic rock that have not been folded by later deformation. The dominant magnitude eigenvector is the mean attitude of this distribution
2. **Uniaxial Girdle:** this distribution is typically the result of cylindrical folding of a an original planar structure. When poles to this planar structure are plotted, a girdle distribution falls about the great circle that is perpendicular to the hinge of the fold. In this case the two dominant eigenvectors will fall on the girdle great circle. The hinge is the vector perpendicular to this great circle, and this of course will be the attitude of the small eigenvector. This eigenvector is the best-fit hinge attitude.
3. A third possibility is that the three eigenvectors have different values but are different magnitudes. This equivocal situation may be caused by uniformly random data, or conical folding. If conical folding is suspected, the Ramsay method should be used to evaluate the structure.

V. Goodness of Fit Measures

- a) In addition to calculating the least-squares geometry, one must also quantify the "goodness of fit" and whether or not the data is normally distributed about the least-square geometry.
- b) If the data are normally distributed about the least-square geometry a standard deviation calculation can be used to quantify a confidence region about the least-squares fit. For example, if data from a point cluster can be demonstrated to be normally distributed about the least-squares vector then a conical confidence with an apical angle of four standard deviations (± 2 standard deviations) should include approximately 95% of present and future measurements. The equation for the standard deviation is listed below:

$$S = \sqrt{\frac{\sum_{i=1}^n (\theta_i - \theta_{ideal})^2}{n - 1}}$$

where θ_i is the actual angular arc between the axis of the geometry and the i th data vector, and θ_{ideal} is the angular arc between the surface of the least-squares geometry and the axis of the least-squares geometry, measured in the same plane as θ_i . The variable "n" is the number of data elements. For a vector, cylindrical, and conical fit θ_{ideal} is equal to 0, 90, and the cone apical angle degrees respectively. To calculate the angle theta between any two vectors the following relationship may be used:

$$\cos(\theta_{ij}) = (\cos(\alpha_i))(\cos(\alpha_j)) + (\cos(\beta_i))(\cos(\beta_j)) + (\cos(\gamma_i))(\cos(\gamma_j))$$

where θ_{ij} is the angle between the two vectors i and j that have directional angles $(\alpha_i, \beta_i, \gamma_i)$ and $(\alpha_j, \beta_j, \gamma_j)$.

- c) R^2 (*Coefficient of Determination*) can be calculated to measure the degree to which the covariance of the data is explained by the least-squares geometry. The value of R^2 ranges from 0.0 (no relationship) to 1.0 (perfect relationship). An R^2 value of 1.0 could only be attained if every data vector falls perfectly on the least-squares surface. A purely random data set would produce an R^2 value of 0 for a least-squares plane or cone. It is not possible to calculate R^2 for a least-squares vector because there is no surface fit to the data. The equation for the calculation of R^2 is listed below:

$$R^2 = \frac{\sum_{i=1}^n (\theta_E)_i^2}{\sum_{i=1}^n (\theta_O)_i^2}$$

where θ_E (“expected”) represents the angle between the geometric mean vector of the data set (equivalent to a least-squares vector fit) and the fit surface (cylindrical or conical) measured in the plane that contains the mean vector and axis of the least-squares surface. The θ_O (“observed”) angle is the arc between the geometric mean and the “*ith*” data vector measured in the plane common to both. Note that if the least-squares conical or cylindrical surface passes perfectly through each data vector the value of $(\theta_E)^2$ and $(\theta_O)^2$ are equivalent, therefore, R^2 would equal unity. As the deviations of data vectors from the fit surface become larger, the denominator of the equation becomes larger, causing R^2 to become lower in value. The θ_E angle is equivalent to 90 degrees if the fit surface is cylindrical, the $\frac{1}{2}$ apical angle if the fit surface is conical. R^2 cannot be calculated for a vector fit.

d) A test for normal distribution can be accomplished with the χ^2 statistic. A full discussion of this method is beyond the scope of this text (see Davis, 1992), however, the below equation is given as for reference:

$$\chi^2 = \sum_{i=1}^n \left(\frac{[(\theta_O)_i - (\theta_E)_i]^2}{(\theta_E)_i} \right)$$

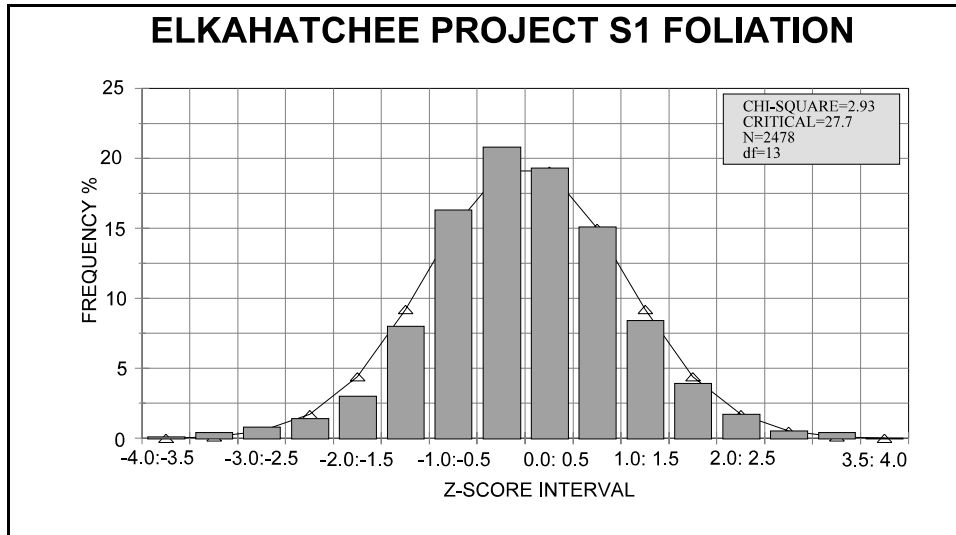


Figure 9-2: Example of data set that is normally distributed about a least-squares cylindrical surface according to the chi-square statistic.

θ_O and θ_E are defined as described above. As can be verified from the above equation, as the data deviate from a normal distribution the χ^2 statistic grows larger in magnitude. Standard statistics texts contain tables for this statistic that requires the *degrees of freedom* (df) and the desired confidence level. Since data must be converted to *z scores* before plotted as a frequency distribution, the df is the number of categories (bars) on the frequency histogram minus the number of calculated values necessary for the z scores (mean + standard deviation = 2). If the χ^2 statistic is larger than the critical value given in tables, the data fail the test and cannot be considered a normal distribution. The Figure

9-2 example above is an example of a data set that is normally distributed about a least-squares geometry according to the χ^2 statistic of 2.33 (critical value = 27.7, same as used by the stereonet analysis program NETPROG).

EXERCISE 9A: Stereograms and Statistical Techniques

Refer to your class notes relating to statistical analysis of orientation data.

In this lab you will apply statistical techniques discussed in class to actual orientation data. In the below exercise you will use NETPROG to calculate best-fit statistics on 3 separate problems demonstrating vector, cylindrical and conical data distributions. A measure of the “goodness-of-fit” will be calculated and plotted, and a χ^2 statistic will be used to evaluate whether the data distribution is “normal” or “non-normal”.

Problem 1. Below is a set of measured orientations of a linear platinum-bearing zone collected by a mining company. The company wants to sink a mine shaft along the zone and therefore needs an average orientation determined from the data. You are a geologist employed by a consulting firm and your supervisor has assigned to you the task of analyzing the data for the mining company. The head of the mining company - a person well versed in statistics, but not in structural geology - informs you that contouring the data and "eyeballing" an average orientation is not good enough; he wants a statistical determination of the average orientation of the data, a statistical measure of the goodness of fit, and a measure of whether or not the data is normally distributed. Determine the best-fit (mean) vector to this data set using the eigen vector method, and determine the standard deviation about the best-fit vector in degrees. Plot the data as points on a stereonet, and plot the position of the best-fit vector. Assuming that the data is normally distributed about the mean vector, plot on the stereonet a "cone of confidence" that should contain over 90% of the data (i.e. 2 standard deviations) if the data is normally distributed.

S 40 W 39	S 38 W 25	S 36 W 21	S 23 W 30
S 42 W 41	S 36 W 29	S 33 W 26	S 17 W 35
S 36 W 47	S 34 W 35	S 32 W 29	S 15 W 34
S 29 W 50	S 28 W 38	S 26 W 30	S 26 W 22
S 44 W 29	S 25 W 40	S 24 W 34	S 25 W 25
S 36 W 33	S 21 W 43	S 19 W 38	
S 34 W 41	S 15 W 44	S 36 W 15	
S 28 W 43	S 28 W 36	S 32 W 20	
S 17 W 50	S 23 W 38	S 30 W 25	
S 33 W 38	S 14 W 40	S 26 W 28	

Make sure the following appear on the stereonet for Problem 1:

Data and Eigen vectors plotted correctly (13 points)

Orientation of best-fit vector (plotted as a dot plus cross and list the plunge and bearing) (10 points):

2-S.D. cone of confidence plotted and listed, χ^2 listed (plotted as a conical surface on stereonet) (10 points)

Problem 2. Below are strike and dip measurements of bedding taken from a cylindrical fold system. Determine statistically the orientation of the hinge using the Eigen vector method. Determine the standard deviation of the fit relative to the data. Plot the data as poles to bedding, and plot the best-fit hinge on the stereogram. Plot the great circle at 90° to the best-fit hinge. Also plot the pair of conical surfaces that lie at two standard deviations on either side of the least-squares cylindrical girdle– this describes the 95% confidence belt.

N 73 W 64 W	N 45 W 90 E	N 56 W 77 W	N 00 E 50 E
N 21 W 68 E	N 03 E 42 E	N 55 W 74 W	N 15 W 54 E
N 75 W 53 W	N 06 W 56 E	N 54 W 82 W	N 25 W 61 E
N 76 E 43 E	N 17 W 57 E	N 81 W 51 W	N 88 W 51 W
N 83 E 52 E	N 28 W 66 E	N 10 W 50 E	N 69 W 64 W
N 88 E 46 E	N 25 W 72 E	N 16 W 68 E	
N 73 E 49 E	N 81 W 60 W	N 43 W 80 E	
N 70 W 66 W	N 71 W 59 W	N 38 W 76 E	
N 15 W 62 E	N 64 W 65 W	N 33 W 80 E	
N 85 W 58 W	N 66 W 70 W	N 30 W 73 E	

Make sure the following appear on the stereonet for Problem 2:

Data and Eigen vectors plotted correctly (13 points)

Orientation of best-fit cylindrical hinge and girdle great circle (hinge plotted as a dot with a cross and list the plunge and bearing) (10 points):

2-S.D. cone of confidence plotted and listed, χ^2 and R^2 listed (plotted as a conical surface on stereonet) (10 points)

Problem 3. Below are foliation measurements from the eastern Blue Ridge of Alabama. The data come from a terrane that has experienced more than one folding event, therefore, the folding of foliation is conical in nature rather than cylindrical. Plot the data as poles to foliation on the stereonet, and calculate the best-fit conical axis. Determine the standard deviation of the conical surface. Plot and label the conical surface and cone axis on the stereonet. Plot the pair of conical surfaces that lie at \pm two standard deviations relative to the least-squares conical surface.

Table 3- Foliation attitudes for Problem 3.			
N 38 W 40 W	N 08 E 68 W	N 19 W 62 W	N 06 W 64 W
N 34 W 51 W	N 10 W 78 W	N 01 E 76 W	N 02 W 74 W
N 24 W 53 W	N 54 W 34 W	N 16 W 57 W	N 12 W 68 W
N 12 W 60 W	N 06 E 80 W	N 27 W 56 W	N 19 W 65 W
N 08 W 70 W	N 26 W 64 W	N 29 W 45 W	N 40 W 50 W
N 09 E 75 W	N 07 W 56 W	N 47 W 44 W	
N 17 E 72 W	N 22 E 24 E	N 44 W 37 W	
N 30 E 85 W	N 65 E 18 E	N 31 W 49 W	
N 41 E 80 W	N 88 W 30 W	N 58 W 40 W	
N 14 E 80 W	N 58 W 30 W	N 43 W 32 W	

Make sure the following appear on the stereonet for Problem 3:

Data and Eigen vectors plotted correctly (13 points)

Orientation of best-fit conical hinge and conical small circle (hinge plotted as a dot with a cross and list the plunge and bearing and conical angle) (10 points):

2-S.D. cone of confidence plotted and listed, χ^2 and R^2 listed (plotted as 2 conical surfaces on stereonet) (10 points)

EXERCISE 9B: Stereograms and Statistical Techniques

In this lab you will apply statistical techniques discussed in class to actual orientation data. In the below exercise you will use NETPROG to calculate best-fit statistics on 3 separate problems demonstrating vector, cylindrical and conical data distributions. A measure of the “goodness-of-fit” will be calculated and plotted, and a χ^2 statistic will be used to evaluate whether the data distribution is “normal” or “non-normal”.

Problem 1. Below is a set of measured orientations of a linear platinum-bearing zone collected by a mining company. The company wants to sink a mine shaft along the zone and therefore needs an average orientation determined from the data. You are a geologist employed by a consulting firm and your supervisor has assigned to you the task of analyzing the data for the mining company. The head of the mining company - a person well versed in statistics, but not in structural geology - informs you that contouring the data and "eyeballing" an average orientation is not good enough; he wants a statistical determination of the average orientation of the data, and a statistical measure of the goodness of fit. Determine the best-fit (mean) vector to this data set using the eigen vector method, and determine the standard deviation about the best-fit vector in degrees. Plot the data as points on a stereonet, and plot the position of the best-fit vector. Assuming that the data is normally distributed about the mean vector, plot on the stereonet a "cone of confidence" that should contain over 90% of the data (i.e. 2 standard deviations).

Table 1- Mineralized zone linear attitudes for Problem 1.			
N 51 E 45	N 59 E 42	N 66 E 35	N 57 E 43
N 48 E 42	N 60 E 36	N 64 E 25	N 62 E 33
N 47 E 35	N 59 E 36	N 84 E 53	N 64 E 27
N 48 E 25	N 58 E 29	N 79 E 48	N 73 E 40
N 58 E 52	N 59 E 24	N 71 E 40	N 71 E 43
N 60 E 44	N 62 E 35	N 68 E 30	
N 53 E 41	N 75 E 51	N 70 E 28	
N 55 E 33	N 70 E 46	N 79 E 38	
N 68 E 50	N 67 E 48	N 76 E 40	
N 65 E 45	N 69 E 38	N 53 E 23	

Make sure the following appear on the stereonet for Problem 1:

Data and Eigen vectors plotted correctly (13 points)

Orientation of best-fit vector (plotted as a dot plus cross and list the plunge and bearing) (10 points):

2-S.D. cone of confidence plotted and listed, χ^2 listed (plotted as a conical surface on stereonet) (10 points)

Problem 2. Below are strike and dip measurements of bedding taken from a cylindrical fold

system. Determine statistically the orientation of the hinge using the Eigen vector method. Determine the standard deviation of the fit relative to the data. Plot the data as poles to bedding, and plot the best-fit hinge on the stereogram. Plot the great circle at 90° to the least-squares hinge. Also plot the pair of conical surfaces that lie at two standard deviations on either side of the least-squares cylindrical girdle– this describes the 95% confidence belt.

Table 2- Bedding attitudes for Problem 2.			
N 44 W 80 E	N 13 W 71 W	N 20 W 77 W	N 48 E 43 W
N 17 E 54 W	N 57 E 37 W	N 24 W 86 W	N 30 E 42 W
N 42 W 72 E	N 32 E 44 W	N 23 W 78 W	N 18 E 49 W
N 64 W 59 E	N 26 E 44 W	N 47 W 70 E	N 53 W 69 E
N 65 W 66 E	N 10 E 50 W	N 42 E 44 W	N 38 W 83 E
N 55 W 64 E	N 11 E 57 W	N 23 E 55 W	
N 68 W 64 E	N 54 W 83 E	N 09 W 61 W	
N 39 W 85 E	N 39 W 78 E	N 03 W 58 W	
N 29 E 55 W	N 33 W 84 E	N 02 W 59 W	
N 51 W 76 E	N 35 W 89 E	N 06 E 56 W	

Make sure the following appear on the stereonet for Problem 2:

Data and Eigen vectors plotted correctly (13 points)

Orientation of best-fit cylindrical hinge and girdle great circle (hinge plotted as a dot with a cross and list the plunge and bearing) (10 points):

2-S.D. cone of confidence plotted and listed, χ^2 and R^2 listed (plotted as a conical surface on stereonet) (10 points)

Problem 3. Below are foliation measurements from the eastern Blue Ridge of Alabama. The data come from a terrane that has experienced more than one folding event, therefore, the folding of foliation is conical in nature rather than cylindrical. Plot the data as poles to foliation on the stereonet, and calculate the best-fit conical axis using Ramsay's method. Determine the standard deviation of the conical surface. Plot and label the conical surface and cone axis on the stereonet. Plot the pair of conical surfaces that lie at \pm two standard deviations relative to the least-squares conical surface.

Table 3- Foliation attitudes for Problem 3.			
N 24 W 23 E	N 64 E 39 W	N 70 W 29 E	N 85 E 34 W
N 42 W 27 E	N 89 W 40 E	N 76 E 46 W	N 79 E 44 W
N 60 W 25 E	N 03 E 20 E	N 77 W 27 E	N 89 W 42 E
N 84 W 30 E	N 70 E 51 W	N 58 W 35 E	N 74 W 36 E

Table 3- Foliation attitudes for Problem 3.			
N 83 E 35 W	N 64 W 36 E	N 47 W 24 E	N 33 W 28 E
N 65 E 46 W	N 86 E 26 W	N 16 W 27 E	
N 54 E 45 W	N 76 E 50 E	N 06 W 21 E	
N 44 E 62 W	N 59 E 38 E	N 45 W 20 E	
N 34 E 66 W	N 33 E 31 E	N 00 W 25 E	
N 60 E 52 W	N 18 E 24 E	N 05 E 18 E	

Make sure the following appear on the stereonet for Problem 3:

Data and Eigen vectors plotted correctly (13 points)

Orientation of best-fit conical hinge and conical small circle (hinge plotted as a dot with a cross and list the plunge and bearing and conical angle) (10 points):

2-S.D. cone of confidence plotted and listed, χ^2 and R^2 listed (plotted as 2 conical surfaces on stereonet) (10 points)