Abstract: Wittgenstein's *Tractatus* carefully distinguished the concept *all* from the notion of a truth-function, and thereby from the quantifiers. I argue that Wittgenstein's rationale for this distinction is lost unless propositional functions are understood within the context of his picture theory of the proposition. Using a model *Tractatus* language, I show how there are two distinct forms of generality implicit in quantified *Tractatus* propositions. Although the explanation given in the *Tractatus* for this distinction is ultimately flawed, the distinction itself is a genuine one, and the forms of generality that Wittgenstein indicated can be seen in the quantified sentences of contemporary logic.
In 1919, not long after he had given Russell a copy of the *Tractatus*, Wittgenstein wrote to Russell,

I suppose you didn't understand the way how I separate in the old notation of generality what is in it truth-function and what is purely generality. A general proposition is a truth-function of *all* propositions of a certain form.¹

The separation of truth-function from pure generality was clearly important to Wittgenstein. He had expressed it in the *Tractatus* at 5.521:

I separate the concept *all* from the truth-function.

If Russell hadn't seen this separation in the notation, the oversight is understandable, not least because the "old notation" that Wittgenstein was using was Russell's own. Indeed, I shall argue that the fact that the notation was not perspicuous in the way that Wittgenstein thought it was reflects a tension in the *Tractatus*’ account of generality. Nonetheless, Wittgenstein's treatment of general propositions did give expression to a distinction among types of generality within quantified propositions that I will claim is justified.

In what follows, I examine how Wittgenstein derives propositional functions, truth-functions, and the general propositions formed from them, within the context of a simplified model language, based on a proposal of John Canfield.² My intention is to make perspicuous how Wittgenstein regarded propositional functions and quantified formulae as emerging from actual, used propositions of a language, and how his doing so

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¹ Wittgenstein 1979, 131.
² Canfield 1972, 200f.
enables us to see distinct forms of generality in quantified propositions. Making this case requires showing how Tractarian propositional functions and quantified formulae are importantly different from the propositional functions and formulae of contemporary logic. The contemporary conception of propositional functions that I have in mind is one traceable to the work of Hilbert and Ackermann. According to it, a propositional function originates as an uninterpreted syntactical object, and it is specified by first classing signs according types, such as sentential variables and predicate variables, and then stipulating rules for the construction of formulas from the typed signs.

I think that applying this conception of propositional functions to the Tractatus distorts the book. For example, consider how Wittgenstein's assertion at 3.333 that “A function cannot be its own argument” fares when we understood the functional sign as a syntactical object along broadly Hilbert-Ackermann lines:

The function sign itself does not declare what can and cannot complete it to make a sentence. If the function sign is thought of strictly as a sign and not as a sentence form (i.e., an implicitly stated formation rule), then the 'x' in 'f(x)' is merely a place marker, showing that some other sign must be placed there in order to make a sentence. (Allaire 1960, 15)

Wittgenstein’s restriction on the arguments taken by functions is used by him to block Russell’s Paradox. But on Allaire’s reading of function signs as first and foremost syntactical objects, nothing blocks “f(f(x))” from being a sentence, contrary to 3.333. And this is just the conclusion Allaire draws in claiming that, "Wittgenstein is led to believe, mistakenly, that the type rule, which is a formation rule, follows from the type

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distinction [the division of signs into classes based upon their syntactical shape] itself"
(ibid, 15).

Allaire's objection presupposes a way of regarding propositional function
expressions in the Tractatus that is not uncommon. ¹ I think, however, that the mistake
that he claims to find in Wittgenstein is an artifact generated by reading the contemporary
notion of a propositional function into the Tractatus. As I shall show, Wittgenstein has
well-grounded reasons for imposing his constraints on function signs, and these appear
when we understand how function signs are propositional variables formed from
significant propositions. But we must see Wittgenstein's ideas about propositional
functions as emerging from framework different from the model-theoretic one that would
develop later. ⁵

Propositional Functions and Satzvariablen

If Wittgenstein's notion of a propositional function was not that proposed by
Hilbert and Ackermann, what was it? To answer this, it is worth first looking at Russell's
notion of a propositional function in the Principia Mathematica, and seeing what
Wittgenstein did -- and didn't -- accept in it. In the Principia, Russell described
propositional functions as expressions containing a variable that become a proposition
upon that variable's being given a fixed determined meaning (ibid, 14). They are
expressed by the carat notation 'ˆx', which distinguishes the propositional function "ˆx is

¹ In Loomis 2005, I look at a variety of other attempts, from Carnap to the present, to read the Tractatus
through the lens of contemporary logic.
⁵ Several recent Tractatus commentators share my desire to resist reading the Tractatus through the prism
of the contemporary, model-theoretic conception of logic. My own understanding of the logic of the
Tractatus is especially indebted to Baker 1988; Varga von Kibed 1993; Ricketts 1996; Hylton 1997; and
Floyd 2002.
hurt" from the "ambiguous" open expression "x is hurt". Russell did not regard
propositional functions as a species of mathematical functions, but to the contrary took
propositional functions as "the fundamental kind of function from which the more usual
kinds of function, such as 'sin x' … are derived." Mathematical functions, which he
called "descriptive functions", were introduced separately. Descriptive functions
"describe a certain term by means of its relation to their argument. Thus 'sin π/2'
describes the number 1." Russellian propositional functions, on the other hand, are not
descriptions of terms but rather are compound, structured entities that share their
structure with the propositions that are their values. One can see in the values “Caesar is
hurt” and “Brutus is hurt” a common shared structure that shows them both as a value of
'x is hurt'; a kinship clearly absent between the descriptive function 'sin π/2' and its value
1. For Russell, the propositional function expressed by 'x is hurt' is thus not a bare, open
syntactical formula, but a structured compound formed from the previously given
propositions that serve as its values.

The *Tractatus* follows this account of propositional functions in an important
respect. Wittgenstein says directly at 3.318 that he conceives of the proposition, as Frege
and Russell do, as a function of the expressions contained in it. And like Russell,
Wittgenstein regards his propositions not as names for objects, but as complexes

6 Russell and Whitehead 1960, 15. Russell thought that 'x is hurt' is an "ambiguous value" of the function
'x is hurt'.
7 Russell and Whitehead 1960, 15. Hylton has shown that the priority propositional functions over other
types of functions is also apparent also from the *PM* definition of non-propositional functions at *20.01. Cf.
9 Russell goes so far as to say that a propositional function is "more complex than its constituents";
meaning by "constituents" the propositions that constitute its values (Russell and Whitehead 1960, 6).
10 Russell was aware of this feature: "[T]he values of a [propositional] function are presupposed by the
function, not vice versa" (Russell and Whitehead 1960, 39). It is because "x is hurt" becomes a proposition
when x is given any fixed meaning that it is a propositional function.
consisting of elements combined in a definite way (3.14). This is essential to his picture theory. In the case of elementary propositions, a proposition's being a complex of elements is required for its being a picture of a possible state of affairs, which is itself a complex of elements (2.0272). As with Russell, Wittgenstein's propositions are thus structured compounds, and this informs his conception of propositional functions.

Wittgenstein's propositional functions are introduced as *Satzvariablen* – "propositional variables". At 3.313, Wittgenstein indicates how a *Satzvariable* is formed, by taking any part of an elementary proposition that contributes to the proposition's sense and changing that part into a variable. The result of this change is "a class of propositions which are all the values of the resulting variable proposition" (3.315). As with Russell, Wittgenstein regards the value of the *Satzvariable* to be determined by the propositions that are its possible values, or as he puts it, by "indicating the propositions whose common mark the variable is" (3.317). And, as with Russell's propositional functions, the *Satzvariable* shares a form with these propositions by presupposing *all* of the propositions in which it can occur (3.311).

Behind this similarity between Wittgenstein’s and Russell’s conceptions of propositional functions there nonetheless lie two important differences. First, Russell freely introduces negation, conjunction, and other truth functions as propositional functions that take propositions as arguments. Wittgenstein rejects this, and indeed at 3.332 claims that propositional functions cannot take propositions as arguments at all. Second, Russell is committed to claiming that truth functions such as negation and

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11 I use "*Satzvariable*" exclusively to designate Wittgenstein's propositional functions. I do not see any difference between the notion of a *Satzvariable* introduced in the 3.3s and the 4.24 idea that the elementary proposition can be expressed as "a function of its names."

12 *Cf.* Russell and Whitehead 1960, 6-7.
conjunction characterize the sense of a proposition, for like all propositional functions, these share a structure with the "aggregations of subordinate propositions" from which they are formed. Thus for Russell, any two instances of that aggregation (values of the function) have some structural commonality. Wittgenstein to the contrary flatly rejects the supposition that truth-functions, which he calls “operations”, might characterize the sense of a proposition (cf. 5.25).

Model Satzvariablen

Wittgenstein's rationale for these claims about propositional functions is carefully grounded in his conception of the proposition, and unintelligible apart from it. This is best seen through the analysis of elementary propositions and the propositional functions formed from them, and the non-elementary and general propositions formed from the elementary ones, in the context of a simple model akin to one proposed by Canfield (1972). The model consists of a world with two different color objects, primary green, named by the symbol “g”, primary blue, named by “b”, and four points of a miniature field, named “p₁”, “p₂”, “p₃” and “p₄”. The points are arranged as follows:

\[
p₁ \ldots p₂ \ldots p₃ \ldots p₄
\]

Concatenating a color name with a point forms an elementary proposition. Thus, “gp₁” says that primary green is at p₁. I shall call the model language used to describe the field “\(L₀\)”.

A qualification is necessary here. Elementary propositions such as “gp₁” in \(L₀\) describe the model world, but they should not be understood as reports of what is visible.

This qualification is made to avoid placing the model's elementary sentences at odds with Wittgenstein's 6.3751 remark that, "the assertion that a point in the visual field ["Gesichtsfeld"] has two different colors at the same time, is a contradiction" (my emphasis). Now, it is essential to elementary propositions in the Tractatus that they be independent (cf. 4.211), as are the elementary facts they represent. Hence “gp₁ & bp₁” must be a consistent proposition stating that both primary green and primary blue are at p₁. There is no tension here with claim made at 6.3751, however, provided that we observe a distinction between two colors being combined at a point, on the one hand, and two colors being co-exemplified by a point, on the other. The latter would occur if a point were visibly green and visibly blue at a single time. The former would occur if we combined green and blue paint at a point. At 6.3751, Wittgenstein excludes only co-exemplification as logically impossible. He does not exclude the possibility that the combination of colors is both possible and expressible as a logical product.¹⁴ As such, the sentence 'gp₁ & bp₁' in the model language \(L₀\) can be understood as a significant proposition expressing the combination, but not the co-exemplification, of two colors at a point. It should thus be understood as part of a possible analysis of visible colors, and not as itself a description of what is visible.

¹⁴ Indeed, that Wittgenstein had countenanced the combination of two colors at a point is clear from his "Remarks on Logical Form", in which he says that he had assumed in the Tractatus that a complete analysis would demonstrate the impossibility of two colors appearing together at a point in the visual field by showing how statements of differences of color would analyze into conjunctions of elementary propositions. Each such proposition would express different degrees of brightness or shade, and the analysis would expose "some sort of contradiction" in the joint assertion of two colors at a point in the visual field; cf. Wittgenstein 1929, 168. As this proposed analysis reveals, Wittgenstein clearly did not intend the Tractatus to exclude the combination of colors at a point, and indeed presupposed the possibility of such a combination, such as that elementary propositions expressing different units of brightness \(b'\) and \(b''\) could be combined, and the combination expressed as a logical product (ibid). Infamously, Wittgenstein acknowledged in the same essay that the combination of two colors cannot be finally analyzed as a logical product, contrary to what Tractatus had assumed. This problem, however, is a defect intrinsic to the Tractatus itself, and is not a feature imposed by the model language \(L₀\).
In $L_0$, the expression 'gp₁' is a *Tractatus* symbol. It is a sign, consisting of perceptible marks, coupled with a significant use, namely, the use specified by the elucidations that I have given above for “g”, “p₁”, and their concatenation.\(^{15}\) The signs “g”, “p₁”, and their concatenation are also *Tractatus* expressions. Expressions are everything essential for the sense of a proposition that propositions can have in common with one another (3.31). “gp₁” can have something in common with, for instance, “gp₂”, “bp₁”, and “bp₂”. We understand this commonality from the elucidations, and grasping it is a necessary condition of understanding these propositions. Put otherwise, not seeing that “gp₁” has something in common with expressions like “gp₂” and “bp₁” entails not understanding “gp₁”. Similarly, not to see that that the expression “g” is a color name and so something that can be at “p₂”, and that “b” is a color name and so something that can be at “p₁”, is to fail to understand “g” and “b” as expressions. As an expression, “g” "presupposes the forms of all propositions in which it can occur. It is the common characteristic mark of a class of propositions" (3.311).

The class of elementary propositions for which “g” is a common characteristic mark in $L_0$ are “gp₁”, “gp₂”, “gp₃”, and “gp₄”. Following 3.312, we can represent the form of this class by a variable, “gy”. Here gy's values are the propositions that contain “g” (*cf.* 3.313). Likewise, we can replace “g” in “gp₁” with a variable to determine another class of propositions thus: “xp₁”. Following Wittgenstein's instruction at 3.315, we can further form the variable expression “xy”, which has as its values all of the color-point propositions. We can still further represent gp₁ by means of the variable “r” (*cf.*, 4.24). In “xy”, we grasp what the substitution instances are for the variables, and in doing so, that they must be distinct. The four variable expressions: “gy”, “xp₁”, “xy”, and “r”,

\(^{15}\) *Cf.* 3.11, 3.263, 3.326.
are examples of Wittgenstein's *Satzvariablen*. *Satzvariablen* expose that an elementary proposition is a function of its names by showing us what elements are expressions, that is, are essential for the sense of the proposition, and what propositions have in common with one another (*cf.* 3.31, 4.24).

These *Satzvariablen* are constructed from meaningful propositions, as we see when we construct them according to Wittgenstein's instructions in the 3.3s. It would be wrong to say that Wittgenstein requires that every *Satzvariable* be so constructed, for at 5.501 he indicates that the description of the variables for a proposition can be given by:

1. “direct enumeration”,
2. “giving a function $f_\alpha$, whose values for all values of $\alpha$ are the propositions to be described”,
3. “giving a formal law, according to which these propositions are constructed.”

The *Satzvariablen* formed from sentences of $L_0$ are of the second form. Such *Satzvariablen* are unlike the propositional functions common in contemporary logic, for they are not formed by first giving independently-specified syntactical schemata, such as “$xy$” or “$Fx$”, and then subsequently assigning an "interpretation" that specifies the possible values such schemata might take. The difference is highlighted by Wittgenstein’s assertion that, "The rules of logical syntax must follow of themselves, if we only know how every single sign signifies" (3.334), and by his dismissal of the possibility of enumerating logical forms a priori as “arbitrary” (5.554-1).

In $L_0$, we can observe that the symbol “$gp_1$” is also a fact, namely the fact that “g” left-flanks “$p_1$”. Seeing “$gp_1$” as a fact, and not simply as an object, a compound name, or a cluster of names, is essential to seeing it as saying that primary green is at $p_1$ (*cf.* 3.1432). Here the left-flanking relation has to be noticed in order for the elementary
proposition to describe a state of affairs. There is, of course, no necessity that the sign “g”
left flanks “p₁” in order to describe this state of affairs. \( L₀ \) involves arbitrary agreement,
as we see when we consider that in a different language \( L₁ \), with a different logical
syntax, we might have expressed that there is primary green at \( p₁ \) by saying “p₁g”. In \( L₁ \),
the Satzvariable \( xy \) must assume different values for its constituent variables, which, per
3.316, are thereby different variables. As Wittgenstein tells us at 3.315, “r” can function
as a Satzvariable for a proposition of either \( L₀ \) or \( L₁ \), since in \( r \) all arbitrary determination,
including the conventions governing the concatenation of expressions, is removed. Yet \( r \)
still determines a class of propositions.

Both “gp₁” in \( L₀ \) and “p₁g” in \( L₁ \) thus share something, which Wittgenstein calls
the "form of representation" (2.17). This form is what any picturing fact must have in
common with what it represents, namely the constraints on possible configurations of
colors and points (cf. 2.15). Unlike the languages of everyday life, whose ambiguity
allows for errors (3.323), the simple languages \( L₀ \) and \( L₁ \) do not allow for the expression
of what is not possible. For instance, in neither \( L₀ \) nor \( L₁ \) is it possible to say anything
illogical like "green is green at \( p₁ \)". This impossibility is embedded in the logical syntax
of each language, and manifests in the elucidations of the primitive signs and the
concatenation relation. And while this constraint on possible formations is common to
both \( L₀ \) and \( L₁ \), it is not stated by either one of them but shown. One does not understand
the possible facts of the model world if one does not see that a color of a color of a point
is not among them. Likewise, one does not grasp either “gp₁” or “p₁g” as depicting the
fact that primary green is at \( p₁ \) unless one also excludes “ggp₁” or “p₁gg” as nonsense.
The claim that an elementary proposition like “gp₁” is a fact is connected with the claim that it shares something in common with what it represents. As with a picture, the concatenation of names in an elementary proposition must mirror the concatenation of objects in a possible fact (Sachverhalt), such that facts are depicted by facts (cf. 2.13-5). Moreover, the elementary proposition must be articulate in exactly those places where the represented fact is articulated; it must possess "the same logical (mathematical) multiplicity" (4.04f.). The names in an elementary proposition, which "go proxy for" (vertreten) their objects in sentences, must have logico-syntactically admissible concatenations in a sentence which mirror the combinatorial possibilities of the objects named. Only thus does a proposition represent (darstellen).\textsuperscript{16}

There is thus an internal reciprocity between a proposition's sense and the fact it represents. Which fact is indicated by a proposition is, of course, determined by what the proposition says. Yet equally, that the proposition says what it does is determined by the possibilities intrinsic to the fact it represents. We cannot, for instance, have a proposition which asserts what is impossible, for Wittgenstein makes it a necessary condition of something's being a proposition that it have truth-possibilities corresponding to the possibilities of the existence or non-existence of possible facts (cf. 4.25, 4.3). This condition is justified by his notion of picturing. A picture of reality must have some conditions of agreement with the world that may or may not obtain (cf. 4.462). A picture that agrees with the world in every case (or no case) is not a proposition but a tautology or a contradiction. By 4.2, the agreement or disagreement of a proposition with these possibilities is the proposition's sense. So a sentence to which there corresponds no possible fact is \textit{eo ipso} senseless and not a proposition. And a sentence in which any sign

\textsuperscript{16} For a further discussion of Wittgenstein's \textit{vertreten/darstellen} distinction, see Ricketts 1996, 74f.
exhibits combinatorial possibilities that do not correspond to combinatorial possibilities had by the object for which the sign goes proxy corresponds to no possible fact.

Consider this in the context of the attempt to say something illogical by substituting propositions as values of variables in Satzvariablen. Russellian propositional functions require type-theoretic restrictions on such substitutions such that one cannot predicate a first-order function of a propositional function of type 1, for instance. Wittgenstein regards such restrictions as unnecessary. We see at once in $L_0$ that "$xy$" cannot be a value for "$y$" in "$xy$". The pseudo-symbol “$x(xy)$” does not predicate a color of a point. A colored point, such as $gp_1$, is a constellation of objects – a fact. There is no issue of predicking a color of a fact. Likewise, the significant symbol “$gp_1$” is a fact. There is no issue of a fact being chained (verketten, cf. 4.22) together with an object, like the name “g”, to form another fact. Indeed, no fact can right-stand any substitution instance of “$x$” in ‘$xy$”, because no fact can right-stand anything. There is no symbol “$x(xy)$” possible in $L_0$ or its extensions that is a configuration of objects such as is required for picturing (cf. 2.031-15). Here it is important to distinguish sign from symbol (cf. 3.32-.326). There is no significant use for the perceptible marks (the sign) “$x(xy)$” in $L_0$ by which it is a significant fact (a symbol). This point generalizes beyond $L_0$; no propositional function can take another propositional function as its argument on pain of its ceasing to be a chaining of objects (names) into a fact.

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17 See Russell and Whitehead 1960, 48-55.
18 They are also inexpressible. Consider Russell’s statement that no first-order function contains a function as an apparent variable in Russell 1908, 165. If $ϕ$ is a first-order function, the propositional function "$q(q(x))$" is not false but nonsense, as is its negation. Yet if one attempted to say that no first-order function contains a function as an apparent variable within the language of Russell’s type-theory, then any statement expressing this restriction would have to contain a variable in the argument position of a first-order function that ranged over first-order functions. But the type theory prohibits just such a variable.
19 No extension of $L_0$ can place an object to the left of the class of facts indicated by this variable. Wittgenstein made a similar point in his dictations to Moore. Cf. Wittgenstein 1979, 116.
These considerations form the grounds for Wittgenstein's rejection of type theory at 3.332:

No proposition can say anything about itself, because the propositional sign cannot be contained in itself (that is the whole “theory of types”).

Wittgenstein calls a property "internal" if it is unthinkable that its object not possess it (4.123). The representational form of a significant *Tractatus* expression is an internal property in this sense, since it is intrinsic to that expression's making a contribution to a proposition as a picture of a state of affairs. In $L_0$'s world, the internal property of colors, which requires that they be predicated of points and not of facts, shows itself by means of the internal property of the *propositions* of $L_0$, which requires that no fact stands to the right of an object. This is the mirroring of internal properties by internal properties that Wittgenstein demands (*cf.* 4.124). Here the syntactical constraints on expressions are not something specified in terms of criteria of sentence composition that might ignore the application of those expressions.

I think that this is behind Wittgenstein's insistence at 4.126 that "formal concepts cannot, like proper concepts, be presented by a function." Rather, formal concepts, or concepts of internal properties, are signified by *Satzvariablen* (4.127, 4.1272).

“Function”, for example, signifies a formal concept. A sentence of the form "$f$ is a function" is not an expression about a mathematical or logical object in the way in which one of the form "$x$ is an even number" is. The sentence "$f$ is a function", even in the common, mathematical sense of “function”, presupposes the application of *Satzvariablen* such as “$f(x) = y$”. The sentence "$f$ is a function" does no more than elucidate the role of “$f$” in such an application. Contravening Wittgenstein's requirement at 4.1272 that we
represent functions as variables and not functions would require attempting to understand "f is a function" as itself a function "g(f)". Once this move is made, special restrictions are required to prevent the application of g to itself to yield "g(g)", and thereby Russell's paradox. But from the perspective of Wittgenstein's picture theory such restrictions are unnecessary because functions are presented as variables, not as functions. The variable “f(x)” expresses a form with its restrictions built in, for "the functional sign already contains the prototype of its own argument and it cannot contain itself" (3.333).

Tractatus Operations and Compound Propositions

As I noted above, Wittgenstein and Russell regarded propositional functions as constructed from propositions, and as distinct from “descriptive functions” such as mathematical functions. Wittgenstein assigned a fundamental importance to this difference. He expressed it by distinguishing between operations, which exhibit the features of mathematical functions, and propositional functions (5.25). Wittgenstein's use of the word "truth-function" in the Tractatus, while carefully explained by him, invites misunderstanding if it is understood in the contemporary sense. Wittgenstein's truth functions are neither Fregean functions, nor functions of names as elementary propositions are. Rather, truth-functions are the results of operations, as Wittgenstein makes clear at 5.234: "The truth-functions of elementary propositions are results of operations which have the elementary propositions as bases" (cf. also 5.3).

Wittgenstein defines an operation at 5.23 as "that which must happen to a proposition in order to make another out of it." Operations generate propositions from other propositions by being "the expression of a relation between the structures of [the
Wittgenstein's dicta that an operation does not characterize either the sense of a proposition (5.25), or its form (5.241), but rather only indicates differences between forms of propositions (5.24). Furthermore, Wittgenstein's general form of the proposition requires that every proposition be the result of some one truth operation on elementary propositions (5.3). The one truth-operation is joint denial, which Wittgenstein indicates by the operation sign “N”. The application of the N-operator to a single elementary proposition \( p \) returns its negation \( \sim p \). The application of \( N \) to two propositions \( p, q \) returns their joint denial, \( \sim p \& \sim q \) (5.51). Wittgenstein thus requires that conjunction and negation be operations (cf. 5.2341). These constraints must be met by elementary propositions, since they are the bases of operations. What features of elementary propositions allow operations to satisfy these constraints? The answer is disarmingly simple.

Recall that for Wittgenstein elementary propositions are independent of one another.\(^{20}\) This independence insures that the conjunction of any two elementary propositions can be treated truth-functionally. Independence requires that there be no logical import internal to the structure of an elementary proposition, for if there were then whether one elementary proposition is true might follow from (or contradict) another's being true – a possibility that Wittgenstein denies (cf. 4.211, 5.134). Since they are independent, the joint assertion of any two elementary propositions \( p, q \) is equivalent to their logical product. Conjunction is thus intrinsic to the structure of elementary propositions as logically independent pictures.

\(^{20}\) The following account of conjunction follows that of Baker 1988, 98f. It is also sketched in Hintikka and Hintikka 1986, 106-9.
Wittgenstein also regards negation as intrinsic to the structure of elementary propositions. I noted above that the sense of a proposition is its agreement or disagreement with reality, that is, its bipolarity. The possibility of denial is, as Wittgenstein says, pre-judged in the affirmation of a proposition (5.44). It is pre-judged because there is no possibility of picturing unless there are conditions of agreement and disagreement with the world.\(^{21}\) To be able to say that a proposition “\(p\)” is true or false, we must be able to call “\(p\)” true (cf. 4.063). Negation and conjunction are thus operations internal to the picturing function of elementary propositions.

This account of how Wittgenstein's \(N\)-operator is built-up from the structural features of elementary propositions reveals how "an operation shows itself in a variable" (5.24). The variable required to show the operation \(N\) is the \textit{Satzvariable} “\(p\)” (or “\(r\)”, etc.). No further logical multiplicity is required once it is understood that this \textit{Satzvariable} is only formed from a genuine proposition with the internal properties of bipolarity and independence that are required for the \(N\)-operator.

Given that every proposition is the result of the one truth-operation \(N\) applied to elementary propositions, how is the picture theory to be extended to non-elementary propositions? This has seemed to some to be less than clear. Michael Kremer, for instance, has objected that on this account it is "not obvious that a conjunction of pictures is also a picture."\(^{22}\) Kremer asks us to consider an example in which we:

"conjoin" a picture of Tom standing to the left of Paul and a picture of Tom standing to the right of Mary by drawing Tom standing between Paul and Mary. Now suppose that we "negate" a picture by literally "using it in an opposite

\(^{21}\) A point that Wittgenstein emphasizes in the case of propositions at 4.462, and in the case of pictures in general at 2.221-.222.
\(^{22}\) Kremer 1992, 419. Kremer addresses his objection to the account of joint negation given by Hintikka and Hintikka, \textit{op cit.}
sense," by turning it upside down. How can we conjoin the denials of the two simple pictures in our example? If we "merge" the upside-down pictures of Tom standing to the left of Paul and Tom standing to the right of Mary, we get an upside-down picture of Tom standing in between Paul and Mary, which is the denial of the conjunction of the two pictures, rather than the conjunction of their denials.\(^{23}\)

Kremer's objection rests on his example, which is intended to show the implausibility of regarding the operators as arising simply from the independence and bipolarity of elementary propositions. Kremer thinks that "merging" the negation of two simple (propositional) pictures of: (A) Tom to the left of Paul and, (B) Tom to the right of Mary gives a new picture, (C) Mary between Tom and Paul.

This apparent consequence is generated by Kremer's suggestion that we negate the pictures by inverting them. Kremer is correct that inversion can be used to accomplish the logical operation of negation, in which case inverting the pictures and the operator sign \(~\) are the same \textit{Tractatus} symbol. However, we must be careful to recognize that in this use, the negation of a picture by inversion is not the same as the assertion of the inverted picture. Negating picture (A) by inverting it says that Tom does not stand to the left of Paul. It does \textit{not} say that Paul stands to the left of Tom, despite the fact that the inverted picture might, in a \textit{different} form of use, be made to depict this. Likewise, the negation by inversion of the two pictures (A) and (B) is not equivalent to the assertion of what is depicted when the pictures are held upside-down and read as if they were themselves pictures of a new state of affairs. Yet this is precisely what Kremer must do to get picture (C); it is only on the assumption that the negation of (A) puts Paul to the left

\(^{23}\) Kremer 1992, 419.
of Tom, and the negation of (B) puts Mary to the right of Tom, that Kremer is able to claim that the conjunction of the negation of (A) and (B) yields (C).

Wittgenstein clearly indicates that truth-tables are propositional signs (4.44, 4.442). They are not definitions of logical operators over bare syntactical objects but rather are, like all propositional signs, facts that stand in a projective relation to the world. The truth-table for the conjunction of \( p \) and \( q \) is thus itself a propositional picture, complete with representational form, truth-poles, and so on. Its negation is clearly distinct from the conjunction of the negation of \( p \) and the negation of \( q \). But regarding truth-tables as propositional signs for non-elementary propositions prohibits any account by which non-elementary propositions can be formed by the merging or melting together of the elements of elementary propositions. Wittgenstein can use only the operator \( N \) for the construction of non-elementary propositions. Non-elementary propositions are therefore not formed by chaining together names, as elementary propositions are, but rather by using operations to expose the internal relations among propositions. This presupposes that in non-elementary propositions, the names of complexes or of the relations among complexes do not "go proxy for" (vertreten) new objects and relations, but instead disappear on analysis.

Consider this in the case of the complexes Tom and Paul, and the left-of relation between them, as expressed by the statement "Tom is to the left of Paul". Following Wittgenstein's instruction at 2.0201, the propositions expressing their relations "can be resolved into a statement about their constituents and into the propositions that describe the complexes completely". The statement in the analysis will be a truth-function

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24 Although Kremer suggests that "Tom is to the left of Paul" is his picture-language might be an elementary proposition, it cannot be, as Canfield has shown in a similar context. See Canfield 1972, 214.
(product of an operation) of elementary propositions, each of which will in turn be a concatenation of names of simple objects (4.22f.).

To see how this can work, suppose we extend the model world of $L_0$ to include complexes and relations among them.\textsuperscript{25} The extension consists of a field of six points, arranged as follows:

\[
p_1 \ldots p_2 \ldots p_3 \\
p_4 \ldots p_5 \ldots p_6
\]

There are also two complex objects. One is denoted by the name “Tom”, and consists of two vertically-arranged contiguous green points in the field. For simplicity, assume that diagonally opposite points are not contiguous, and no color complexes overlap a point. Another complex is denoted by the name “Paul”, and consists of two vertically arranged contiguous blue points. Let $L_2$ be an extension of $L_0$ that describes this world, and that includes the additional names “Tom” and “Paul”, as well as the relation-sign “left of”, and the logical operations of conjunction and negation (“&” and “~”). “Tom left of Paul” is a propositional sign in $L_2$ which states that two contiguous green points stand to the left of two contiguous blue points, as for instance when $p_1$ and $p_4$ are green, and $p_2$ and $p_5$ are blue.

Following 2.0201, “Tom left of Paul” analyzes into two conjuncts, each of which decomposes into elementary propositions. The first conjunct, $\alpha$, is a statement about the constituents of the complexes mentioned in “Tom left of Paul” which gives us a complete characterization of those complexes and how they could be arranged:

\[
\alpha
\]

\textsuperscript{25} The following example expands on Canfield's "blau/grün" example in his 1972, 351f. I have expanded the field from that given for $L_0$ in order to avoid the trivializing result of having only one possible left-standing relation for complexes.
Since complexes cannot overlap, the complete characterization of the complexes mentioned in “Tom left of Paul” requires the explicit exclusion of blue and green at a point.

“Tom left of Paul” has a second conjunct, $\beta$, which "describes the complexes completely". It is:

$$\beta = [(gp_1 \& \neg bp_1) \& (gp_4 \& \neg bp_4) \& (bp_2 \& \neg gp_2) \& (bp_5 \& \neg gp_5)] v [(gp_2 \& \neg bp_2) \& (bp_5 \& \neg bp_5) \& (bp_3 \& \neg gp_3) \& (bp_6 \& \neg gp_6)] v [(gp_3 \& \neg bp_3) \& (gp_6 \& \neg bp_6) \& (bp_2 \& \neg gp_2) \& (bp_5 \& \neg gp_5)].$$

The three disjuncts in $\beta$ describe the three possibilities in which Tom is left of Paul. For example, the first disjunct places Tom at points $p_1$ and $p_4$, and Paul at points $p_2$ and $p_5$.

The "indeterminateness" shown by $\beta$s having three disjuncts is an inherent feature of any proposition in which an element signifies a complex (cf. 3.24).

Upon analysis, we see that there is nothing for which “left of” goes proxy in “Tom left of Paul”. “Left of” in $L_2$ is not a *Vertreter*. While “left of” makes a contribution to the sense of any proposition in $L_2$ in which it appears, its doing so does not require that we go beyond the operations of conjunction and negation and the internal properties of the simples named by “b”, “g”, “$p_1$”, and so on.\(^{26}\)

\(^{26}\) Infamously, the color-exclusion problem exposes that this type of analysis cannot work in every case. *Cf.* Wittgenstein 1929.
Quantification in the Tractatus

*Satzvariablen* in the *Tractatus* are general. They range over the class of their substitution instances, which class is not given independently of a particular *Satzvariable*, but is rather determined by the possible substitutions within the meaningful proposition from which the variable is formed. Quantificational generality has as a necessary condition the generality carried by *Satzvariablen*. Wittgenstein expresses this by stating that it is peculiar to a symbolism of generality that it refer to a logical prototype (5.522), and that the generality symbol occurs as an argument (5.523). We see this, for example, in the *Satzvariable* “xp₁”, in which generality is expressed by the variable (argument position) “x”. This variable refers to a logical prototype, namely all of the propositions of this form. Nothing about this kind of generality requires the truth-function, as Wittgenstein clearly sees (cf. 5.521).

Quantificational generality introduces the truth-function, which is operator *N*, on top of the generality of *Satzvariablen*. Consider for instance the *Satzvariable* “xp₁” in *L₀*. Let “ξ” denote all of its values, viz., “bp₁” and “gp₁”. Then following 5.52, \( N(\xi) = \neg(\exists x)xp₁ \), "\( N(\xi) \)" is, as noted above, the application of the joint-negation operation to the values of ξ, viz., “¬bp₁ & ¬gp₁”. The range of the values of ξ is set by the *Satzvariable*, which itself requires apprehending the application of elementary propositions in *L₀* to assert facts about its world. Here we can see how "all logical operations are already contained in the elementary proposition" (5.47). A grasp both of the range of the variable “xp₁” required for the construction of a logical product, and of the joint negation operator \( N \) are already implicit in understanding the proposition “gp₁” from which “xp₁” is formed.
Consider next the non-elementary sentence “Tom left of Paul” in $L_2$. From this proposition we can form the Satzvariablen “$w$ left of Paul”, and “$w$ left of $z$”. As with the Satzvariablen constructed from “gp$_1$”, the range of the variables is constrained by the sense of the propositions from which the Satzvariable “$w$ left of $z$” is constructed. The substitution instances for “$w$” and “$z$” are restricted to complexes, but there is no need to mention this restriction because there is no genuine elementary proposition, such as “gp$_1$ left of $b p_2$”, from which the Satzvariable could be formed.\(^{27}\)

In the Satzvariable “$w$ left of Paul”, the values are given by a list in which every complex is mentioned that might stand to the left of Paul in the model world (it would include, among other things, $\beta$). Here two possible cases must be distinguished: one in which a name like “Paul” signifies a type that may be multiply instantiated, and another in which it signifies an individual particular. As I introduced it, “Paul” is functioning as the name of a type. Following Wittgenstein's assertion that the Law of the Identity of Indiscernibles is at most contingently true (5.5302), the set of possibilities indicated by “$w$ left of Paul” must also include the three possible situations in which one Paul, understood as a type, stands to the left of another, numerically distinct Paul.

The denial of the Identity of Indiscernibles alters how we understand a given propositional function because it widens the possible substitution instances for variables in propositional functions formed from non-elementary propositions. However, I do not think that its denial is essential to the Tractatus' account of quantification. If, contrary to 5.5302, we wish to name numerically distinct individuals that cannot be multiply instantiated, then the analysis reveals that no numerically distinct complex can possibly

\(^{27}\) This latter is not a Tractatus proposition since it fails the bipolarity constraint on propositions. See Canfield 1972, 214.
stand to the left of itself, for any such complex reduces to a collection of simples whose internal properties insure different substitution instances for the variable positions in “\(w\) left of \(z\)”. For instance, if we denote by “Paul\(_1\)” any vertically arranged Paul configuration occupying point \(p_1\), so that \(\text{Paul}_1 = b_{p_1} \& b_{p_4}\), then Paul\(_1\) is never left of Paul\(_1\). To see this, suppose for a \textit{reductio} that “Paul\(_1\) left of Paul\(_1\)” is a proposition. Then on its analysis we are met with a contradiction, since in attempting to give complete characterization of this sentence akin to the \(\alpha\) component of “Tom left of Paul”, we would at once be met with the sentence, "\((b_{p_1} \& \neg b_{p_1} \& b_{p_4} \& \neg b_{p_4})\)” (similarly with all of the other disjuncts in \(\alpha\)). “Paul\(_1\) left of Paul\(_1\)” thus fails the bipolarity condition on propositionhood. As a consequence of the fact that “Paul\(_1\) left of Paul\(_1\)” is a pseudo-proposition, the \textit{Satzvariable} formed from “Paul\(_1\) left of \(z\)” is different from that formed from “Paul left of \(z\)” The former prohibits the substitution of Paul\(_1\) for “\(z\)”, while the latter does not.

We can once again represent the values of the \textit{Satzvariable} “\(w\) left of Paul” by “\(\xi\)”, and apply the operator \(N\) to it. Doing so gives us the joint denial of these values. Unlike functions, operations can take their own results as arguments (\(5.251\)), so applying \(N\) again to this result gives us \(N(N(\xi))\), which is the logical sum of the \textit{Satzvariable}’s values. For simplicity, we can represent the logical sum of a set of arguments with “\(\Sigma\)”, and the logical product of a set of arguments with “\(\Pi\)”\(^{28}\). Then “\(\Sigma(w) w\) left of Paul)” states that there is something to the left of Paul (the subscripted “\(w\)” indicates the scope of the “\(\Sigma\)” operator in front of it). From this proposition we can form another \textit{Satzvariable} thus: “\(\Sigma(w) w\) left of \(z\)””. Here “\(z\)” ranges over any object that can left-stand any substitution instance of “\(w\)”. We can take the logical product of these \(z\)-values, and

\(^{28}\) This notational simplification, including the restriction of variables to the ‘\(\Sigma\)’ and ‘\(\Pi\)’ signs below, is due to Ricketts, "Generality and Logical Segmentation in Frege and Wittgenstein", unpublished.
represent them as: \( \Pi(z \Sigma(w \text{ w left of } z)) \), which in modified Russelian notation corresponds to \( (z)(\exists w)(w \text{ left of } z) \). Similar constructions allow for all the other quantification possibilities, including all mixed quantifications.\(^{29}\)

Quantifier notations such as \( \Pi(z \Sigma(w \text{ w left of } z)) \), or equivalents formed with the \( N \)-operator, appear to require conventions governing the variable-names within the scope of the operators. This is required by Wittgenstein's elimination of the identity sign, and by his insistence that identity of object be expressed by identity of sign (5.53). Yet Wittgenstein did not tell us how these restrictions might appear.\(^{30}\) Why not? Because, I suggest, he believed that the construction of \( \text{Satzvariablen} \) from genuine propositions would make the relevant restrictions clear, and that that attempting to state these

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\(^{29}\) My analysis of this sentence here follows Ricketts (\textit{ibid}), and Varga von Kibéd,s 1992. As von Kibéd illustrates in detail, R. Fogelin's claim (in Fogelin 1982), that there is a "fundamental error" in the \textit{Tractatus} which prohibits the construction of certain mix-quantifier formulae such as \( (z)(\exists w)(w \text{ left of } z) \), is incorrect and ignores the variability possible within Wittgenstein's \( \text{Satzvariablen} \). Against Fogelin, von Kibéd shows that we can negate such formulas as \( \text{faa, fba, fac} \) using operator \( N \), and further construct the joint negation of sets of these sentences (von Kibéd 1992, 89f; cf. also Jacquette 2001). Thus, we can apply \( N \) to sets of formulas like: \( \{\text{faa, fba, fac, ...}\}, \{\text{fba, fbb, fbc, ...}\}, \{\text{fca, fcb, fcc, ...}\}, ..., \text{to yield: } \{\sim\text{faa} \& \sim\text{fba} \& \sim\text{fbb} \& \sim\text{fbc}, ..., \sim\text{fca} \& \sim\text{fcb} \& \sim\text{fcc}, ..., \} \). Applying \( N \) to this set in turn yields: \( (\text{faa v fbb v fbc} v ... \& (\text{fba v fbb v fbc} v ...) \& (\text{fca v fcb v ...}) \& ... \). This is equivalent in the \textit{Tractatus} to \( (z)(\exists x)fzx \), which is precisely one of the formulas that Fogelin denies can be constructed.

In the case of \( L_2 \), a modification would be necessary to insure that sentences like \( (z)(\exists w)(w \text{ left of } z) \)” aren’t necessarily false in virtue of there being no points to the left of \( p_1 \) and \( p_6 \). One such modification would involve understanding the points as forming a loop, so that \( p_1 \) and \( p_6 \) would appear to the left of \( p_3 \) and \( p_4 \). With such a modification, \( (z)(\exists w)(w \text{ left of } z) \)” would be analyzable along the same lines that von Kibéd has proposed, with the \( w \) and \( z \) variables ranging over possible cases of complexes like Tom and Paul left-standing one another.\(^{30}\)

For some possible interpretations of how the required restrictions might appear, see Hintikka 1956, 228-9, and Floyd 2002, 324-7. Ricketts proposes the following restrictions for his \( \Sigma \) and \( \Pi \) notation that I have used above:

1. Variable-names with embedded scopes must be distinct.
2. If one variable lies within the scope of a second, then the first variable cannot simultaneously take as a value the same name that the second does.
3. No variable may take as a value any name within its scope.

conditions by means of a genuine *Tractatus* propositions would be neither possible nor necessary. We see, without being told, what the *Satzvariable* ranges over.\(^{31}\)

*The Limits of Showing*

In his post-*Tractatus* philosophical career, Wittgenstein readily employed the notion of a linguistic rule as something wholly or partially constitutive of the meaning of an expression. Yet the notion of a rule plays no real role in the *Tractatus*. There are indeed brief references to "rules of logical syntax", but the upshot is that these are *consequent upon*, rather than constitutive of, the proper functioning of signs.\(^{32}\) Prior to writing the *Tractatus*, Wittgenstein surely understood the possibility of attempting to frame rules for a language such as Russell's type restrictions. His considered response was that such rules are useless pseudo-propositions. By the same token, the sentences of the *Tractatus* itself are not formulations of rules. They are, as Wittgenstein tells us at 6.54, "elucidations" (*erläuterungen*). Their function is in one sense akin to the elucidations given above for expressions such as “gp₁” in \(L₀\). Elucidations are not a matter of laying down rules governing the application of signs, because propositional symbols like “gp₁” do not appear as such until their use in picturing is grasped (compare 3.263). This grasping requires understanding what is pictured by “gp₁”. Yet once “gp₁” is understood, any attempt to formulate rules for its use is rendered redundant. The

\(^{31}\) As Floyd has pointed out, this leads to important differences between Wittgenstein and the contemporary understanding of propositional functions like “\((x)(y)(fxy & fyx) \supset (x)fxx\)”. For Wittgenstein this is not a truth of logic but is a significant proposition telling us something about the relation \(f\). Cf. Floyd 2002, 325-6.

\(^{32}\) Cf. 3.334, quoted above. Compare also 3.325 and 6.124, which convey the sense that the rules of logical syntax are *determined* by the "nature of the essentially necessary signs" of a language.
sentences of the *Tractatus* are equally redundant for any one who has grasped the proper use of the expressions of their language.

There is a tension here, however. As we have seen, many of Wittgenstein's *Satzvariablen* emerge from the use of propositions as pictures by "giving a function \( fx \), whose values for all values of \( x \) are the propositions to be described" (5.501). The function in this case is a propositional function, a *Satzvariable* formed from actual, used propositions. How is this model to be applied to a proposition such as, "All men die before they are 200 years old"? This proposition involves a generalization over collections that are neither surveyable in the way that \( L_0 \)'s world is, nor constructible by means of formal series. This proposition therefore must be the logical product of all values of the *Satzvariable* "\( x \) died before he was 200 years old", where this in turn is formed from statements such as "Socrates died before he was 200 years old", "Plato died before he was 200 years old", and so on. Yet the totality of the possible values of this variable are nothing that we can plausibly be said to see, as we can be said to see all the possible values of "\( xp_1 \)" in \( L_0 \). Wittgenstein must say that (as he later put it), "though its terms aren't enumerated here, they are capable of being enumerated (from the dictionary and the grammar of language)".³³

Wittgenstein indirectly gave expression to this idea in his *Tractatus* claim that the world can be completely described by completely general propositions without coordinating any name with a definite object (5.526). Such a set of propositions would delimit all and only the possible states of affairs of the world (5.5262). But without knowing all the terms of the logical sums and products that the *Tractatus* holds that such propositions are equivalent with, our understanding of such a set of propositions is left a

mystery. It is no use to promise here that we *could* provide an enumeration of the relevant instances *were* we to perform a complete analysis. For Wittgenstein's whole account of the *Satzvariablen* and the truth-operations formed with them gets its grip from genuine propositions used to describe possible facts.

It is possible that Wittgenstein was misled by the use of simple, finite examples. In his "Criticism of my former view of generality", he reported that:

it is correct that $(\exists x).q x$ behaves in some ways like a logical sum and $(x).q x$ like a product; indeed for one use of words "all" and "some" my old explanation is correct, -- for instance for "all the primary colours occur in this picture" or "all the notes of the C major scale occur in this theme". But for cases like "all men die before they are 200 years old" my explanation is not correct.\(^{34}\)

In his *Notebooks* of 1914, Wittgenstein provided a small example illustrating the claim, later made at *Tractatus* 5.526, that the world could be described by completely general propositions without coordinating signs with names. He first described the world as one that "consisted of the things A and B and the property F, and that F(A) were the case and not F(B)" and then described it again by means of the general propositions:

"$(\exists x,y). (\exists \phi). x \neq y. x \phi \land y \phi. u. \phi z. \supset u. z = z^\prime", \"(\exists \phi). (\psi). \psi = \phi^\prime", \text{ and } \"(\exists x,y). (z). z = x \lor z = y.\"^ {35}\) One here sees that the general propositions describe the world given the prior description of it as consisting of A, B, F, and so on. Given that we have, as it were, a God's-eye view that allows us to see before us *all* of the objects, along with *all* of their combinatorial possibilities, we assent to the general descriptions as complete, because we see how to construct the *Satzvariablen* and thereby form the required logical sums and

\(^{34}\) Wittgenstein 1978, 268.

\(^{35}\) Wittgenstein 1979, 14.
products. The general propositions then delimit the range of possibility only because we see from the initial description the range of *Satzvariablen* like "φx". From such a perspective, it is indeed possible to describe the world without coordinating any name with an object, as Wittgenstein says. We might then imagine that the case of ordinary language must be similar, and thus that while we cannot survey all of the values of the *Satzvariable* beforehand, some such totality of values must nonetheless be present (compare 5.5562).

For anyone in the world however, logic must, as Wittgenstein put it, "have contact with its application" (5.557). This "contact" in the context of general propositions includes the specification of the actual propositions from which the *Satzvariable* required for the quantified proposition is constructed. I think that Wittgenstein's use of Russell's notation for generality conceals this; nothing about the *Principia*'s use of the quantifiers seems to require of us that we be able to survey the possible substitution instances of a variable beforehand. Our understanding of "(x).φx" seems to be exhausted by saying, with Russell, that it denotes "all values of φx". But by the *Tractatus*' lights it is not so exhausted; we must know how the *Satzvariable* "φx" was formed before we can understand what is indicated by "all values".

*Generality and Logical Form*

That Wittgenstein's conception of generality cannot be fully squared with the existence of general propositions for which we cannot provide the required analysis does not mean that his conception has nothing to offer us. For there are different senses of generality in logical formulas, and the differences are akin to those Wittgenstein was
trying to elucidate with his separation of the generality of logical form from quantificational generality.  

Consider a quantified formula of contemporary model-theoretic logic, such as “(∀x)(Fx ⊃ Fx)”. It is general in two ways. On the one hand, there is the general applicability of the formula to a collection of independently specifiable instances. On the other hand, there is the generality of the form of a logical construction, which is expressed by arbitrary instances or schematic variables. In the first type of generality, the formula “(∀x)(Fx ⊃ Fx)” is general by being generally applicable to the objects of its domain, which are specifiable independently of the formula itself. This is the type of generality that expressed by the quantifier “(∀x)”. The generality of the form of the logical construction on the other hand, is not expressed but is rather shown by its being a form of possible formulae that all of its instances, such as:

(*) Fa ⊃ Fa, Fb ⊃ Fb, Fc ⊃ Fc, etc.,

have in common. This form is not expressed by the quantifier but rather is presupposed by the entire quantified formula, as we see when we compare the following two sentences:

(1) All the equations (*) have the form Fx ⊃ Fx.

(2) All the equations (*) are validities.

Statement (1) is vacuous in a way that I think is akin to the way that Wittgenstein had thought the sentences of the Tractatus were. The vacuity of (1) lies in the fact that one

36 The following paragraph is inspired by Sören Stenlund's analysis of the equation "a + b = b + a", and the forms of generality contained in it, in Stenlund 1990, 158-9.

37 I'm here assuming an "objectual" interpretation of the quantifier. On a substitutional interpretation, the quantifier expresses that all of the substitution instances of the formula are valid. In this case, all of the substitution instances must be understood independently of what the quantifier says, similarly to what I say here about the objectual case.
cannot understand what is meant by the expression “All the equations (*)” without understanding that sentence (1) is true. On the other hand, recognizing the validity of all the equations (*) is not a precondition of understanding what is meant by “All the equations (*)” in statement (2). It would make sense, for instance, to ask of someone that they prove the truth of (2) by showing that “(∀x)(Fx ⊃ Fx)” holds in all models. But there could be no such proof of the truth of (1); to understand the “etc.” in (*) is to be able to produce or recognize equations of the appropriate form. Understanding the generality of the form of ‘(∀x)(Fx ⊃ Fx)” presupposes a recognition of (*) as a partial list of its instances. This generality of the form is not stated by the quantifier, but is instead a condition of its general applicability.

Wittgenstein's *Tractatus* articulated this distinction and brought it to the fore. By separating the concept all from the quantifier, the *Tractatus* worked to elucidate the fact that apprehending a quantified sentence presupposes apprehending a form of use of expressions that is already general. Wittgenstein's observation of the distinction was to survive the demise of the *Tractatus* project, although it would have to find a new rationale. Finding that rationale would involve a return to the notion of elucidations, and would lead Wittgenstein to the idea that, contrary to what the *Tractatus* had maintained, explanations, such as ostensive definitions, are in fact meaningful rules for the use of expressions.

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38 Consider for instance Wittgenstein's later claim that the infinity of a number series is given by the rules for a number system rather than by classes in his 1975, 161f., his continued dissatisfaction with the Frege-Russell notation for generality when applied to ordinary language (cf. 1978, 265-7; 1979b, 227-9), or his claim that a proposition about all propositions or functions is "a priori an impossibility: what such a proposition is intended to express would have to be shown by an induction" (cf. 1975, 150).

39 Thus for instance, the ostensive definition of something comes to be treated as "a rule for translating from a gesture language into a word language." Wittgenstein 1978, 88.
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