



UNIVERSITY OF SOUTH ALABAMA

GY 302: Crystallography & Mineralogy

Lecture 2: More Symmetry Operations, Bravais Lattices

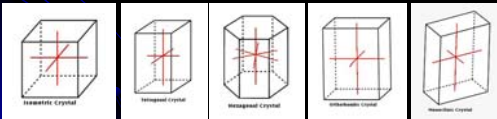
Last Time

- ❖ A long winded introduction
- ❖ Introduction to crystallography
- ❖ Symmetry

Crystal Systems

The 6 (or 7) Crystal Systems.

Crystal System	Axes	Angles between axes	Mineral examples
1 Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90^\circ$	Halite, Galena, Pyrite
2 Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90^\circ$	Zircon
3a Hexagonal	$a_1 = a_2 = a_3 \neq c$	$\alpha = \beta = 120^\circ, \gamma = 90^\circ$	Apatite
3b Trigonal	$a = a_1 = a_2 \neq c$	$\alpha = \beta = 120^\circ, \gamma = 90^\circ$	Quartz, Calcite
4 Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^\circ$	Aragonite, Staurolite
5 Monoclinic	$a \neq b \neq c$	$\alpha = \beta = 90^\circ, \gamma \neq 90^\circ$	Gypsum, Orthoclase
6 Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^\circ$	Plagioclase

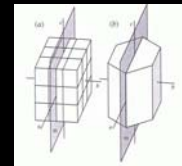


Crystal Systems

In order to fully understand how crystals are "put together", We need to reconsider symmetry.

In the last lab, we examined 2 symmetry operations:

- 1) Reflection (mirror planes)

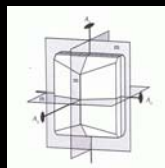


Crystal Systems

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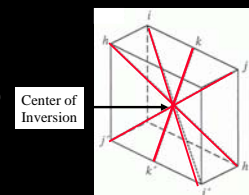
- 1) Reflection (mirror planes)
- 2) Rotation (2, 3, 4, 6 fold axes)



Crystal Systems

There are two other operations that we did not have a chance to discuss at the start of lab 1.

- 1) Reflection (mirror planes)
- 2) Rotation (2, 3, 4, 6 fold axes)
- 3) Inversion



Crystal Systems

- 1) Reflection (mirror planes)
 - 2) Rotation (2, 3, 4, 6 fold axes)
 - 3) Inversion
 - 4) Translation
- Basic symmetry operations
- 5) Screw Rotation
 - 6) Glide Rotation
 - 7) Rotoinversion
- Compound symmetry operations

Today's Agenda

1. Rotoinversion
2. Translational Symmetry
3. Bravais Lattices

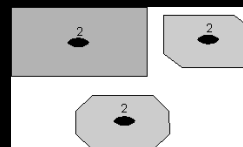
Lecture 2: More Symmetry

Symmetry in Crystals

Rotation axes:

2-fold Rotation Axis - If an object appears identical after a rotation of 180° , that is twice in a 360° rotation, then it is said to have a 2-fold rotation axis ($360/180 = 2$).

Note that in these examples the axes we are referring to are imaginary lines that extend toward you perpendicular to the page or blackboard. A filled oval shape represents the point where the 2-fold rotation axis intersects the page.

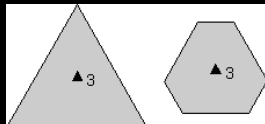


<http://www.cartage.org.lb>

Symmetry in Crystals

Rotation axes:

3-fold Rotation Axis - Objects that repeat themselves upon rotation of 120° are said to have a 3-fold axis of rotational symmetry ($360/120 = 3$), and they will repeat 3 times in a 360° rotation. A filled triangle is used to symbolize the location of 3-fold rotation axis.

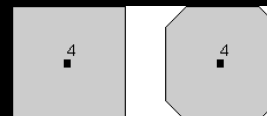


<http://www.cartage.org.lb>

Symmetry in Crystals

Rotation axes:

4-fold Rotation Axis - If an object repeats itself after 90° of rotation, it will repeat 4 times in a 360° rotation, as illustrated previously. A filled square is used to symbolize the location of 4-fold axis of rotational symmetry.

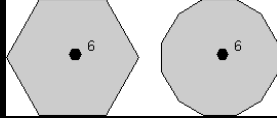


<http://www.cartage.org.lb>

Symmetry in Crystals

Rotation axes:

6-fold Rotation Axis - If rotation of 60° about an axis causes the object to repeat itself, then it has 6-fold axis of rotational symmetry ($360/60=6$). A filled hexagon is used as the symbol for a 6-fold rotation axis.

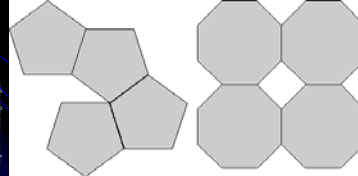


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Symmetry in Crystals

Rotation axes:

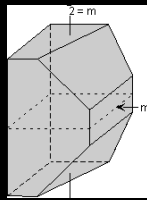
Although objects themselves may appear to have 5-fold, 7-fold, 8-fold, or 12-fold rotation axes, these are not possible in crystals. The reason is that the external shape of a crystal is based on a geometric arrangement of atoms and they must be able to *completely fill in space* in order to exist in nature (Think unit cells or floor tiles).



<http://www.cartage.org.lb>

Symmetry in Crystals

Rotoinversion: A combination of rotation with a center of inversion.



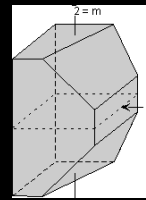
<http://www.cartage.org.lb>

Symmetry in Crystals

Rotoinversion: A combination of rotation with a center of inversion.

2-fold Rotoinversion - The operation of 2-fold rotoinversion involves first rotating the object by 180° then inverting it through an inversion center.

This operation is equivalent to having a mirror plane perpendicular to the 2-fold rotoinversion axis.

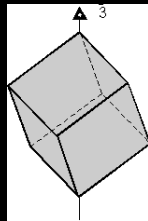


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Symmetry in Crystals

Rotoinversion: A combination of rotation with a center of inversion.

3-fold Rotoinversion - This involves rotating the object by 120° ($360/3 = 120$), and inverting through a center. A cube is good example of an object that possesses 3-fold rotoinversion axes.



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Symmetry in Crystals

Rotoinversion: A combination of rotation with a center of inversion.

4-fold Rotoinversion - This involves rotation of the object by 90° then inverting through a center.

Note that an object possessing a 4-fold rotoinversion axis will have two faces on top and two identical faces upside down on the bottom, if the axis is held in the vertical position.

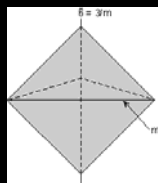


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Symmetry in Crystals

Rotoinversion: A combination of rotation with a center of inversion.

6-fold Rotoinversion - involves rotating the object by 60° and inverting through a center. Note that this operation is identical to having the combination of a 2-fold rotation axis perpendicular to a mirror plane.



<http://www.cartage.org.lb>

Symmetry in Crystals

Translation: Repetition of points by lateral displacement.

Consider 1 dimensional translations:

Symmetry in Crystals

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Symmetry in Crystals

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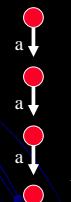
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Symmetry in Crystals

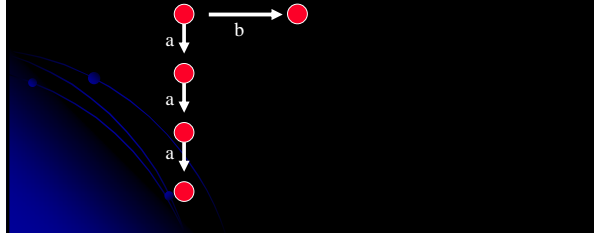
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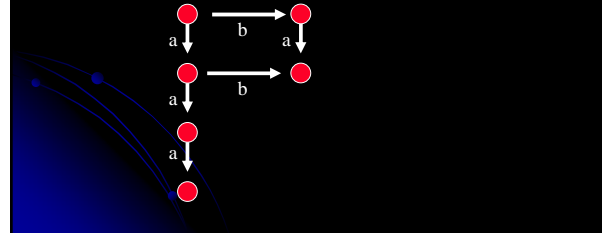
Symmetry in Crystals

Translation: Repetition of points by lateral displacement.
Consider 2 dimensional translations:



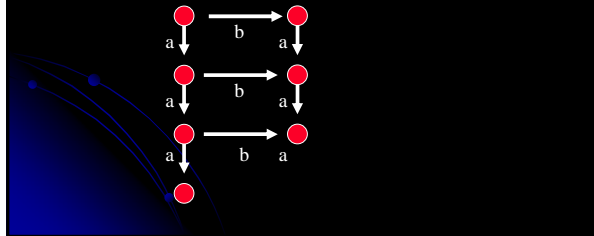
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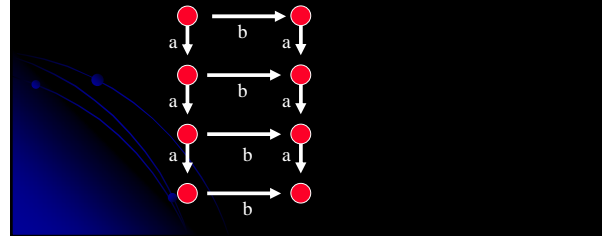
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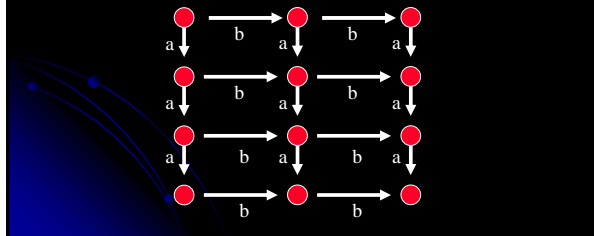
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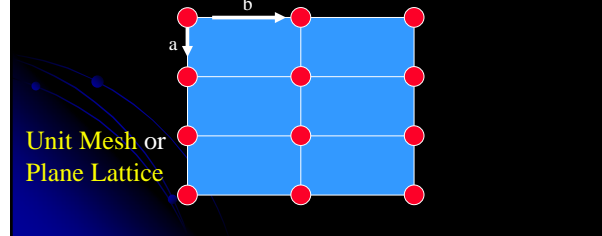
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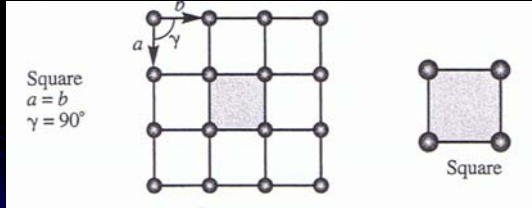


Symmetry in Crystals

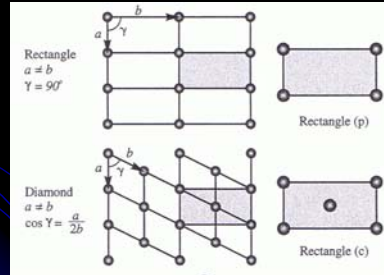
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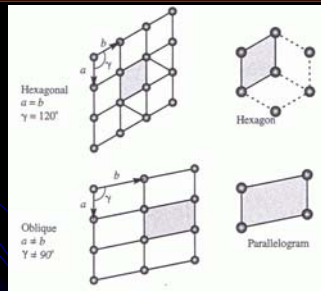
Symmetry in Crystals



Symmetry in Crystals

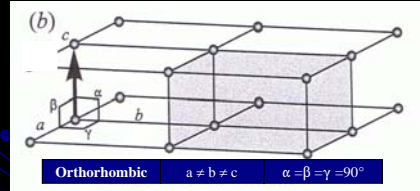


Symmetry in Crystals



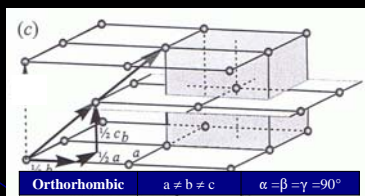
Symmetry in Crystals

Consider 3 dimensional translations:



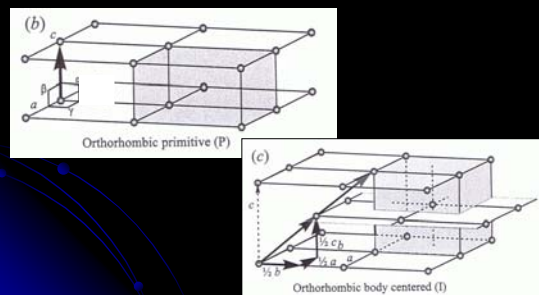
One of 14 possible 3D or Bravais Lattices

Symmetry in Crystals

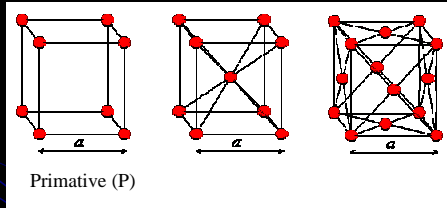


Another possible 3D or Bravais Lattices

Symmetry in Crystals

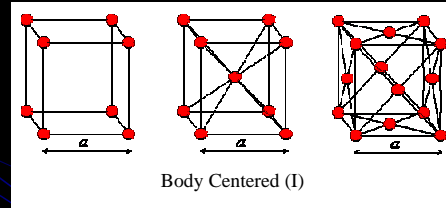


Bravais Lattices



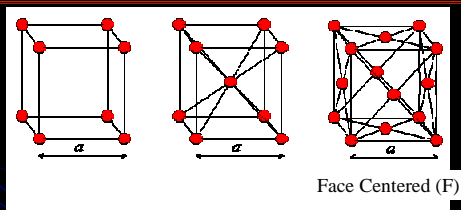
Bravais Lattices in the Isometric System

Bravais Lattices



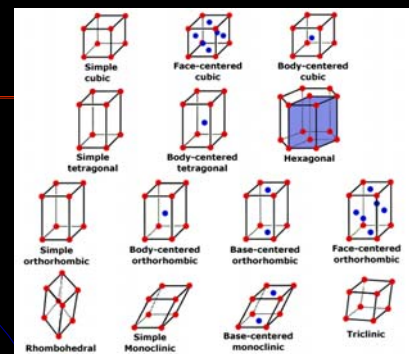
Bravais Lattices in the Isometric System

Bravais Lattices



Bravais Lattices in the Isometric System

The 14 Bravais Lattices

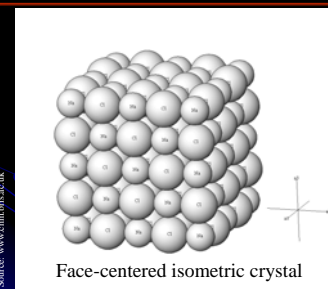


Bravais Lattices

Bravais Lattices in crystals are more or less synonymous with **unit cells** as used by chemists...

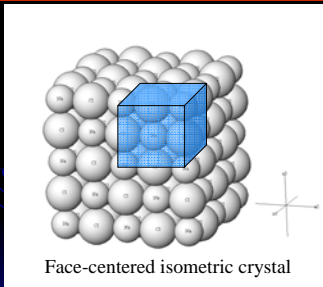
... but unit cells are defined as the smallest repeating unit that forms an infinite crystalline material

Unit Cells



Source: www.dhmi.bham.ac.uk

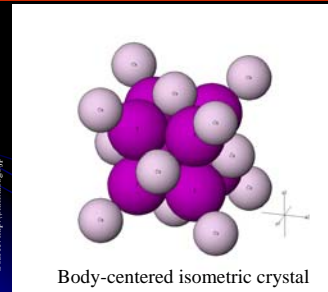
Unit Cells



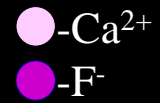
NaCl
(Halite)



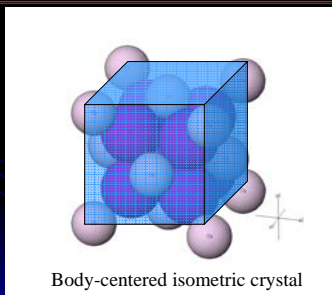
Unit Cells



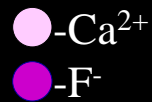
CaF₂
(Fluorite)



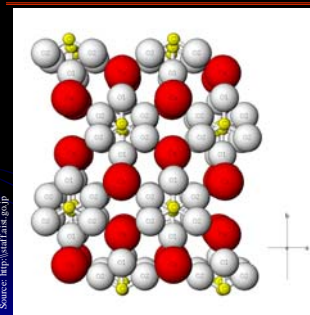
Unit Cells



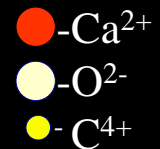
CaF₂
(Fluorite)



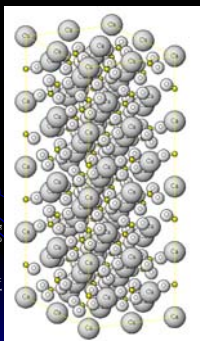
Unit Cells



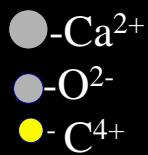
CaCO₃
(Aragonite)



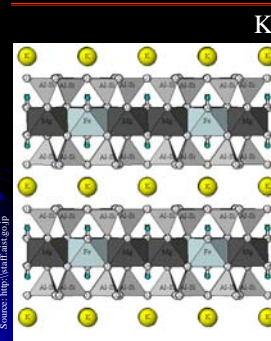
Unit Cells



CaCO₃
(Calcite)



Unit Cells



K(Mg,Fe)₃AlSi₃O₁₀(OH,F)₂
(Biotite)

Today's Homework

1. Finish lab 1 crystals

Next Week

1. Point Groups, Miller Indices
 2. First lab quiz (Isometric & Hexagonal crystals),
more wooden models
- 